STAT 552 Homework 2

Due date: In class on Thursday, September 15, 2005

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5. In a branching process we have

$$P(s) = P_{Z_{j,k}}(s) = as^2 + bs + c \tag{1}$$

with $a > 0$, $b > 0$, $c > 0$ and $P(1) = 1$.

   (a) Compute the extinction probability π.

   (b) Give a condition for sure extinction.

   (c) What is the underlying distribution of offsprings $Z_{j,k}$?

6. Let S_n be a simple random walk, i.e., $S_0 = 0$ and $S_n = S_{n-1} + X_n$, where X_n are i.i.d. Bernoulli random variables which assume the value 1 with probability p and -1 with probability $q := 1 - p$.

   Recall the definition of the first hit of $S_n = 1$, i.e., $N^+ = \inf\{n \geq 1 \,:\, S_n = 1\}$. Similarly, define N^- to be the first hit of $S_n = -1$. Denote by $S_n^\#$ the simple random walk obtained by exchanging p and q and let $N^\#$ denote the first hit of $S_n^\# = 1$. Finally, let N_0 denote the first return to zero of S_n, i.e., $N_0 := \inf\{n \geq 1 \,:\, S_n = 0\}$.

   (a) Show that $N^\#$ and N^- have equal distributions.

   (b) Compute the pgf of N_0. Hint: Consider the two cases $X_1 = \pm 1$ and express the distribution of N_0 in terms of those of N^+ and N^-. This will lead to a formula of P_{N_0} in terms of P_{N^+} and $P_{N^\#}$ which are known from class.

   (c) Compute $P[N_0 < \infty]$.

   (d) Compute $\mathbb{E}[N_0]$. Hint: consider all cases $p > q$, $p = q$, $p < q$ carefully.

7. Consider a simple branching process with $P(s) = q + ps$. Let T denote the time of extinction, i.e., $T = \inf\{n \geq 1 \,:\, Z_n = 0\}$.

   (a) Express the event $\{T = n\}$ in terms of the events $\{Z_k = 0\}$.

   (b) Express $P[T = n]$ in terms of $P(s)$.

   (c) Derive an explicit formula for $P[T = n]$ in terms of p and q. Hint: recursion.