STAT 552 Homework 8

Due: Tuesday, November 22, 2005

Instructor: Dr. Rudolf Riedi

25. (Age and excess life)

    Let N_t be a pure renewal process. Let $A(t)$ be the age of the current item, and $B(t)$ its excess life time as usual. Also, set as usual

    $$F_0(x) = \frac{1}{\mu} \int_0^x (1 - F(u))du. \tag{1}$$

    Recall the renewal equation of the tail probability $P[B(t) > x]$ as well as Blackwell's key renewal theorem, saying that $U * z(t) \to \frac{1}{\mu} \int_0^\infty z(u)du$ under appropriate assumptions.

    (a) Using Blackwell's theorem show that $P[B(t) \leq x] \to F_0(x)$ as $t \to \infty$.

    (b) Similarly, show that $P[A(t) \leq x] \to F_0(x)$.

    Still with the same setting fix now two positive numbers x and y and define

    $$Z(t) := P[A(t) > x, B(t) > y]. \tag{2}$$

    (c) Write a renewal equation for $Z(t)$.

    (d) Compute $\lim_{t\to\infty} Z(t)$.

26. (Markov versus Renewal process)

    Let $\{S_n\}_{n\geq 0}$ denote a *stationary* renewal sequence with inter-arrival times $\{Y_n\}_{n\geq 1}$ uniformly distributed on $[0, 1]$. Let N_t be the renewal process associated with S_n.

    (a) Compute the initial distribution, i.e., compute $G(x) = P[Y_0 \leq x]$.

    (b) Show that $P[N_{s+1/2} = k | N_s = k, N_{s-2/3} = k] = 0$.

    (c) Show that $P[N_{s+1/2} = k | N_s = k, N_{s-x} = k - 1] \geq P[Y_{k+1} \geq x + 1/2]$.

    (d) Conclude that N_t is not Markov.

    Let N_t denote a homogeneous Poisson process (PRM(dt)). Let the r.v. Λ be independent of $\{N_t(\cdot)\}$ and take the values 1 and 2 with probabilities p and $q = 1 - p$, respectively. Let $M_t = N_{\Lambda \cdot t}$ be the associated mixed Poisson process.

    (e) Show that M_t is not a renewal process.
    Hint: The inter-arrival times are not independent. Compute the joint distributions of the first two interarrival times Y_1 and Y_2 by conditioning on Λ.

    (f) Recall that N_t is Markov. Show that M_t is also Markov.
    Hint: Condition on knowing Λ.