STAT 552 Homework 9

Due: Tuesday, November 28, 2006

(practice test: voluntary, not graded)

Instructor: Dr. Rudolf Riedi

26. (Sums of Poisson Point Processes)

    (a) Let N and M be two PRM in same space E with intensity measures ν and μ, respectively, and independent of each other. Show that $K(A) = N(A) + M(A)$ is again a PRM with intensity $\kappa = \nu + \mu$.
    Hint: Assume that the points of N are $\{X_n\}$ and the points of M are $\{Y_n\}$ and determine first what the points of K are.

    (b) Assume now that N and M are PRMs on the positive real line, i.e., on $E = \mathbb{R}^+$ and that both are homogeneous, i.e., ν and μ are both Lebesgue measure: $\mu(a, b) = \nu(a, b) = b - a$ for $b > a > 0$. You may assume also, that the points of N, M and K are ordered, thus, the output of a renewal process with exponential interarrival times of mean 1, 1 and 1/2, respectively. Explain why this shows that the minimum of iid mean 1 exponential random variables is again an exponential r.v., with mean 1/2.

27. Customer arrival times in a bank form a homogeneous PRM with intensity $\mu(dt) = \alpha dt$ on the interval $[9, 16]$, a subset of $\mathbb{R}^+$. The each require a service time with exponential distribution $\sim \exp(\lambda)$. We assume that service starts immediately at arrival (no waiting time...what a wonderful life). How many clients on average are still being serviced when the bank closes at 4pm,i.e., at time point 16?

    Your solution should be $\frac{\alpha}{\lambda}[1 - \exp(-\lambda(b - a))]$ with $b = 16, a = 9$

28. Let w denote a (deterministic) exponential function, i.e., $w(t) = \exp(-\theta t)$ for $t \geq 0$ and zero else, where $\theta > 0$ denotes a given parameter. Given points $\{S_n\}$ set

    $$X(t) = \sum_{n \geq 01} w(t - S_n).$$

    (a) First, assume the points of N form the renewal points S_n of a renewal process with exponential interarrival times $Y_i \sim \exp(\alpha)$ $(i = 0, ...)$, i.e., $S_n = Y_0 + ... + Y_n$. Derive the renewal equation for $\mathbb{E}[X(t)]$ and solve it.
    Hint: recall the simple form of $\mathbb{E}[N(t)]$ here.

    (b) Assume now that S_n form the points of the PRM N, a homogeneous Poisson Point Process on $\mathbb{R}^+$ with intensity measure $\mu(dt) = \alpha dt$. Compute $\mathbb{E}[X(t)]$ in this case.
    Hint: Condition on knowing $N((0, t]) = k$ and use the order statistics property.

29. Let X_i be i.i.d. Gaussian random variables with zero mean and variance 1. Let ν be a Poisson random variable with mean m. Describe the distribution of the point process $N(\cdot)$ given by

    $$N(A) = \sum_{i=1}^{\nu} \mathbf{1}_{\{X_i \in A\}}$$

    as fully as possible.
    Hint: Set $\mu(A) = \mathbb{E}[N(A)]$. Find first $\mu(\mathbb{R})$, then $\mu(A)$ for arbitrary A.