STAT 331 Homework 1

Due date: In class on Thursday, September 9th, 2004

Instructor: Dr. Rudolf Riedi

General comment: Please send an email to riedi@rice.edu. This would allow you to receive urgent announcements in a more timely manner. The email can be absolutely blank; simply add STAT331 in the subject line.

Notation: for improved readability we denote the complement of the set A by A^c instead of $\overline{A}$.

1. (10 points)

   A and B go fishing. By experience, B catches a fish 80% of the time, both catch a fish 60% of the time, and at least one of them catches a fish 90% of the time.

   (a) Find the (absolute) probability that A catches a fish. (.7)

   (b) Find the conditional probability that B catches a fish given that A catches a fish. (0.857)

   (c) Are the events "A catches a fish" and "B catches a fish" independent? Explain. (no)

2. (10 points) Using the three axioms of probability, prove the following properties of probability.

   (a) (4) If $A \subset B$ then $P[A] = P[B] - P[A^c \cap B]$.

   (b) (2) If $A \subset B$ then $P[A] \leq P[B]$. Hint: Use problem (2a).

   (c) (4) If A and B are independent, then A and B^c are also independent.

3. (10 points) In the following, some number of dice are thrown simultaneously. A "straight" is the event when all numbers from 1 through 6 are observed.

   (a) (4) What is the probability that a "straight" occurs when six dice are thrown simultaneously?

   (b) (6) What is the probability that a "straight" occurs when seven dice are thrown simultaneously?

4. (10 points) A manufacturer is concerned about the quality of his product. Experience shows that on the average, one unit out of 10'000 does not meet specifications. In other words, the event B of a unit being bad has probability $P[B] = 10^{-4}$. This not good enough and so, test are performed to catch the bad units. If the unit is okay, the test is 99.9% effective in certifying that the unit is okay. But if the unit is defective, the test is only 80% reliable, meaning that 20% of the bad units test as okay. Units that test okay are shipped. Denote the event that a unit tests defective by F.

   (a) (6) What is the probability p that an item that is shipped does indeed meet the specifications? Hint: compute $P[F]$ first, the answer.

   (b) (4) Using (a) how many out of $100'000$ shipped units would you "expect" to be defective? (Answer: 2)