STAT 331 Homework 2

Due date: In class on Thursday, September 16th, 2004

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5. (15 points)

   In class we learned what it means for two events to be independent. Generalizing, three events A, B and C are said to be independent if and only if

   (a) $P[A \cap B] = P[A]P[B]$, $P[A \cap C] = P[A]P[C]$, $P[B \cap C] = P[B]P[C]$ and

   (b) $P[A \cap B \cap C] = P[A]P[B]P[C]$

   In this exercise we provide an example where (a) holds, but (b) does not. Thus, those three sets are "not independent". A more precise terminology for such a case (where (a) but not (b) holds) is: the three events are only "pairwise independent" but "not fully independent".

   Consider simplified Roulette (without "zero"): $S = \{1, \ldots, 36\}$, $P[\{n\}] = 1/36$ for all $n \in S$. There are 3 events we are interested in: E are the even numbers, R are the red numbers, and F are the numbers in the first row, i.e.

   $$F = \{1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34\}.$$

   Unlike true roulette let us assume that the red numbers are

   $$R = \{1, 2, 3, 7, 8, 12, 13, 14, 15, 19, 20, 24, 25, 26, 27, 31, 32, 36\}.$$

   In a picture, the red numbers are:

1	4	**7**	10	**13**	16	**19**	22	**25**	28	**31**	34
2	5	**8**	11	**14**	17	**20**	23	**26**	29	**32**	35
3	6	9	**12**	**15**	18	21	**24**	**27**	30	33	**36**

   (a) (4 points) Compute $P[E \cap R]$, $P[E \cap F]$, $P[R \cap F]$, $P[E \cap R \cap F]$.

   (b) (3 points) Compute $P[E \,|\, R]$, $P[E \,|\, F]$, $P[R \,|\, F]$.

   (c) (1 point) Are the events E and R independent?

   (d) (1 point) Are the events E and F independent?

   (e) (1 point) Are the events R and F independent?

   (f) (1 point) Are the events E, R and F independent?

   (g) (4 points) Let us assume that instead of the first row,

   $$F = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36\}$$

   denotes the numbers in the *third* row. Of all the 11 questions in 5a to 5f, exactly two have now a different answer. Which are they, and what are the new answers?

6. (10 points)

   An IC consists of 300 transistors, 40 resistors and 1000 connecting parts. The probability that any transistor is defective is 10^{-6}, the probability that any resistor is bad is 10^{-5}, and the probability that any connection brakes is $2 * 10^{-6}$. Assume that all components must function for a proper operation of the IC and that all component failures are independent events.

   (a) (5 points) Find the probability that an IC does not function properly.

   (b) (5 points) A chip is not working; what is the probability that this is due to a faulty resistor? In a more technical language: Find the probability that a resistor of an IC is bad given that the IC does not operate properly.

7. (5 points)

   During Easter egg hunt, young A usually finds at the most 1 egg. Towards a higher yield, A plans to talk a few uncles into finding eggs on A's behalf. From past experience, an uncle finds an egg with probability $p = .4$. What is the minimum number n of uncles to be enlisted in this task if the probability of them finding all together at least 2 eggs should be at least 95%? (Uncles assume to have fulfilled their task after finding one egg and stop searching.) (Answer: 10)

8. (10 points)

   At the hardware store the nuts and screws get mixed up because customers are careless. A bin which is labelled "#8 screws" contains 100 screws, exactly six of which are of the wrong size (not #8). A person intending to buy 12 screws type #8 randomly takes 12 screws out of this bin.

   (a) (5 points) Approximate (or model) the random drawing as a Bernoulli trial. In this model compute the probability that all drawn screws are of the correct size. (0.4759)

   (b) (5 points) Use now combinatorics to find the exact probability (0.4546)