STAT 331 Homework 5

MID TERM

These problems have higher scores than usual.
You can achieve overall 120 points in this homework as compared to the usual 40 points.
Pay specific attention to being clear, explicit and clean in your solution for full credit.

Due date: In class on Thursday, October 14, 2004

Instructor: Dr. Rudolf Riedi

18. (10 points)

    A biased coin falls on Heads with probability p. The coin is flipped 10 times. Let the random variable M be the number of Heads minus the number of Tails seen in the ten flips. Compute the expected value of M.

19. (20 points)

    A telemarketer calls people on the telephone to make appointments for an insurance salesman. The average number of calls needed to make one appointment is 6. You may assume that the responses are independent from call to call.

    (a) (10 points) The telemarketer makes 6 calls. Compute the probability that he had a least two successful calls.

    (b) (10 points) The telemarketer makes calls until he has two appointments. Compute the probability that the second successful call will occur in at most six calls.

20. (10 points)

    A basketball player who hits 60% of her free throws plays the following game. She takes two shots and then K more shots, where K is the number of hits she makes out of the first two. Thus, she takes two, three, or four shots. Let X be the total number she hits.

    Find the expected value of X.

21. (10 points)

    An airline notices that 5% of passengers with reservations do not show up for their flight. Consequently, the airline accepts 52 reservations for the 50 seats available on a given jet. Compute the probability that all passenger with reservation that show up for this flight have a seat (i.e., no passenger with reservation is left behind).

22. (10 points)

    A family has two children. Assume that all possible combinations $\{(g,g),(b,g),(g,b),(b,b)\}$ are equally likely where (g,b) stands, e.g., for first born is a girl, second born is a boy. What is the conditional probability that both are boys given that the first child was a boy?

23. (25 points)

    Let X denote a geometric variable with success parameter p, i.e., $p_X(k) = q^{k-1}p$. Similarly, let Y be geometric with success parameter u and $p_Y(k) = v^{k-1}u$. Assume X and Y are independent.

    (a) (5 points) Show that $P[X > n] = q^n$.

    (b) (7 points) Show that X has no memory in the sense that

    $$P[X > k + n | X > n] = P[X > k]$$

(c) (8 points) Show that $P[\min(X, Y) > n] = (qv)^n$.

(d) (5 points) Using (c), what kind of distribution does the minimum of two independent geometric random variables have?

You may use the result of (a) in all subsequent questions even if you are not able to derive it. The questions can be answered independently of each other.

24. (15 points)

An urn contains 5 balls. The experiments consists of drawing a ball randomly (any of the five with equal probability), mark the ball, return it into the urn, draw a new ball, mark it if it is yet unmarked, return it, and so on until all balls are marked.

(a) (5 points) At the second drawing, what is the change to draw an unmarked ball? What is the expected number of drawings until an unmarked ball is drawn?

(b) (10 points) What is the expected number of drawings until all balls are marked?

25. (20 points)

Suppose that we want to generate a random number X that is equally likely to be either 0 or 1. All we have at our disposal is a coin that is biased, that is, it lands on Heads with some unknown probability $p \neq 1/2$. Someone suggests the following procedure:

(a) Flip the coin and let Y_1 be the result, Heads or Tails.

(b) Flip the coin again and let Y_2 be the result, Heads or Tails.

(c) If Y_1 and Y_2 are the same, then return to step (a).

(d) If Y_1 and Y_2 are different then set $X = 0$ if Y_2 is Heads and $X = 1$ if Y_2 is Tails.

Argue that the random variable X takes the values 0 and 1 equally likely. (12 points).

Could we use a simpler scheme and flip the coin simply until it shows two different faces in the last two flips? (Hint; this can be solved without solving the first question. Consider the case where $p = .99999$ to get an intuition.) (8 points)