ELEC 535 Homework 4

Due date: In class on Friday, February 21, 2003

Instructor: Rudolf Riedi Rice University, Spring 2003

Problem 4.1 (Optimal Codes)

(a) Find some binary ($D = 2$) prefix free code for the source with probabilities $(1/3, 1/5, 1/5, 2/15, 2/15)$.

(b) Find the optimal prefix free code,i.e., the one with minimal expected length. Hint: optimize the expected length under the boundary condition of the Kraft (in)equality for this source and construct a code with the resulting optimal code lengths.

(c) Is this optimal code also optimal for the source with probabilities $(1/5, 1/5, 1/5, 1/5, 1/5)$? Show your reasoning. Hint: any prefix free code has to satisfy the Kraft inequality.

Problem 4.2 (The game of Hi-Lo)

(a) A computer generates a number X according to a known (in particular also known to the player) probability mass function $p(x)$, $x \in \{1, 2, \ldots, 100\}$. The player asks a question, "Is $X = i$?" and is told "Yes", "You're too high", or "You're too low." He continues for a total of six questions. If he is right (i.e. he receives the answer "Yes") during his sequence, he receives a prize of value $v(X)$ (which is independent of how long it takes to find the answer). How should the player proceed to maximize his expected winnings?

(b) The above doesn't have much to do with information theory. Consider the following variation: $X \sim p(x)$, prize $= v(x)$, $p(x)$ known, as before. But *arbitrary* Yes-No questions are asked sequentially (not only "higher-or-lower-than" questions) until X is determined. Questions cost now one unit each. Show that the expected return is $\sum_x p(x)(v(x) - l(x))$, where $l(x)$ is the number of questions required to determine the object x and depends on the strategy of the player.

(c) Maximize the above expected return to find the optimal strategy $l(x)$.