

STAT 582 Homework 2

Due date: In class on Friday, February 11, 2005

Instructor: Dr. Rudolf Riedi

5. Let X_n have normal distribution with mean 0 and variance σ_n^2 . When is the family $\{X_n\}_n$ u.i.?
6. Suppose $\{X_n\}_n$ and $\{Y_n\}_n$ are two u.i. families defined on the same probability space. Is $\{X_n + Y_n\}_n$ u.i.? Show your argument.
7. Let $\{X_n\}_n$ be a sequence of i.i.d. random variables with mean zero and variance σ^2 . Let $\{a_n\}_n$ be a sequence of real numbers. Set

$$S_n = \sum_{i=1}^n a_i X_i.$$

Prove: $\{S_n\}_n$ converges in $L_2 \iff \sum_{i=1}^{\infty} a_i^2 < \infty$.

Hint: If you choose to work with $\sum_{i=1}^{\infty} a_i X_i$ you need to show first that this r.v. exists and in what space it lies. However, you can solve the problem avoiding this random variable (and thus avoiding some headaches).

8. You may use the following fact:

Let a_n be a sequence of real numbers. Assume that $(a_1 + \dots + a_n)/n \rightarrow 0$. Then $a_n/n \rightarrow 0$.

[If you feel like proving this, note that $(a_1 + \dots + a_n)/n = (n-1)/n \cdot (a_1 + \dots + a_{n-1})/(n-1) + a_n/n$.]

Now let X_n be a sequence of r.v. and assume that $(X_1 + \dots + X_n)/n \xrightarrow{P} 0$. Show that $X_n/n \xrightarrow{P} 0$.

Hint: Show the claim first by replacing convergence in probability with a.s. convergence.