STAT 582 Homework 4

Due date: In class on Friday, March 4, 2005

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11. Here, we establish an extension to the SLLN.

    Assume that $\{X_n\}$ are iid with $\mathbb{E}[X_1^-] < \infty$ and $\mathbb{E}[X_1^+] = \infty$.

    (a) Using Kolmogorov's SLLN show that for any $c > 0$ we have $\liminf_n \bar{X}_n \geq \mathbb{E}[X_n \mathbb{I}_{\{X_n < c\}}]$ almost surely.

    (b) Conclude that $\lim_n \bar{X}_n = \infty$ a.s.

    The extended SLLN says then: If $\mathbb{E}[X_1^-]$ and $\mathbb{E}[X_1^+]$ are not both infinite, e.g., if $\mathbb{E}[X_1]$ is well defined, then $\bar{X}_n \to \mathbb{E}[X_1]$.

12. Here, we establish a simplified version of the Three Series Theorem for positive random variables.

    Assume that $\{X_n\}$ are positive. Show that

    if $\sum_n \mathbb{E}[X_n \mathbb{I}_{\{|X_n| < c\}}]$ converges,
    then $\sum_n \mathrm{var}(X_n \mathbb{I}_{\{|X_n| < c\}}) < \infty$.

    In conclusion: The sum of positive independent r.v. converges almost surely iff the two series (i) and (iii) of the Three Series Theorem converge.

    Hint: Recall that the variance is bounded by the second moment.

13. Let X_n be exponential with parameter λ_n, i.e., $P[X_n > x] = \exp(-x\lambda_n)$. Give sufficient and necessary conditions on the series $\{\lambda_n\}_n$ such that $\sum_n X_n$ is almost surely converging. For which $\alpha \in \mathbb{R}$ does $\sum_n X_n$ converge almost surely when setting $\lambda_n = n^\alpha$?