# ELEC 533 Homework 4

Due date: In class on Friday, September 27, 2002

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14. We return to the simplified Roulette (without "zero"): $\Omega = \{1, \ldots, 36\}$, $P[\{n\}] = 1/36$ for all $n \in \Omega$. There are 3 events we are interested in: $E$ are the even numbers, $R$ are the red numbers, and $F$ are the numbers in the first row, i.e. $F = \{1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34\}$. Unlike true roulette let us assume that the red numbers are $R = \{1, 2, 3, 7, 8, 12, 13, 14, 15, 19, 20, 24, 25, 26, 27, 31, 32, 36\}$:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 4 | **7** | 10 | **13** | 16 | **19** | 22 | **25** | 28 | **31** | 34 |
| **2** | 5 | **8** | 11 | **14** | 17 | **20** | 23 | **26** | 29 | **32** | 35 |
| **3** | 6 | 9 | **12** | **15** | 18 | 21 | **24** | **27** | 30 | 33 | **36** |

Consider the random variables $X$, $Y$ and $Z$ given by:

$$X(\omega) = \begin{cases} 4 & \text{if } \omega \text{ is even} \\ 0 & \text{else} \end{cases} \qquad Y(\omega) = \begin{cases} 7 & \text{if } \omega \text{ is red} \\ 0 & \text{else} \end{cases} \qquad Z(\omega) = \begin{cases} -3 & \text{if } \omega \text{ is in the first row} \\ 0 & \text{else} \end{cases}$$

(a) Compute the marginal distributions of $X$, i.e., compute $P[X = t]$ for all $t \in \mathbb{R}$.

(b) Compute the pairwise joint marginal distributions of $(X, Y)$, i.e., compute $P[X = s \text{ and } Y = t]$ for all $s, t \in \mathbb{R}$.

(c) Compute the full joint marginal distributions of $(X, Y, Z)$, i.e., compute $P[X = s \text{ and } Y = t \text{ and } Z = u]$ for all $s, t, u \in \mathbb{R}$.

(d) Are the random variables $X$ and $Y$ independent, i.e., is it true that for all $s$ and $t$ we have $P[X = s \text{ and } Y = t] = P[X = s] \cdot P[Y = t]$?

(e) Are the random variables $X$ and $Z$ independent?

(f) Are the random variables $Y$ and $Z$ independent?

(g) Are the random variables $X$, $Y$ and $Z$ independent, i.e., is it true that $P[X = s \text{ and } Y = t \text{ and } Z = u] = P[X = s] \cdot P[Y = t] \cdot P[Z = u]$?

15. Assume that $X$ and $Y$ are independent Gaussian random variables with zero mean and variance 1. Compute the distribution of the random variable $Z = \exp(-(X^2 + Y^2)/2)$. Hint: the transformation from Cartesian to polar coordinates goes as $x = r\sin(\phi)$, $y = r\cos(\phi)$, $dxdy = rdrd\phi$.

16. Suppose $X$ and $Y$ are joint r.v.'s with

$$f_{XY}(x, y) = \begin{cases} e^{-y} & x > 0, y > x \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $\iint f_{XY}(x, y)\, dy\, dx = 1$ .

(b) Find $f_X$ and $f_Y$.

(c) Are $X$ and $Y$ independent? Show your reasoning.

17. Let the joint density of the pair of random variables $(X, Y)$ be given by

$$f_{XY}(x, y) = \begin{cases} y\exp(-xy) & \text{if } x > 1 \text{ and } y > 0 \\ 0 & \text{else.} \end{cases}$$

(a) Compute the marginal densities $f_X$ and $f_Y$.

(b) Are $X$ and $Y$ independent? Show your reasoning.

(c) Compute $\mathbb{E}[X]$ and $\mathbb{E}[Y]$.

18. Two pairs of discrete random variables $(U, V)$ and $(X, Y)$ are given via their joint distributions:

$$P[U = u, V = v] = \begin{cases} 1/2 & \text{if } u = 1,\ v = 0 \\ 1/6 & \text{if } u = 1,\ v = 1 \\ 1/12 & \text{if } u = -1,\ v = 1 \\ 1/4 & \text{if } u = -1,\ v = 0 \end{cases}$$

and

$$P[X = x, Y = y] = \begin{cases} 7/12 & \text{if } x = 1,\ y = 0 \\ 1/12 & \text{if } x = 1,\ y = 1 \\ 1/6 & \text{if } x = -1,\ y = 1 \\ 1/6 & \text{if } x = -1,\ y = 0 \end{cases}$$

(a) Show that the marginals are the same, that is $F_X = F_U$ and $F_Y = F_V$.

(b) Which pair is independent?