STAT 650 Homework 5

Instructors: Drs. Dennis Cox and Rudolf Riedi

Due date: Thursday, April 27, 2006
Bring to class or hand in to Dr. Riedi, Duncan Hall 2080

12. Solve Oksendal problem 7.19. Notice the hints given.
13. The mean reverting Ornstein-Uhlenbeck process is the solution X_t of

$$dX_t = (m - X_t)dt + \sigma dB_t.$$

Derive an equation for $\mathbb{E}[X_t]$ and solve it. Show that $\mathbb{E}[X_t] \to m$ as $t \to \infty$.

14. Show that $(X_1(t), X_2(t)) = (\cosh(B_t), \sinh(B_t))$ solves

$$\begin{bmatrix} dX_1 \\ dX_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} dt + \begin{bmatrix} X_2 \\ X_1 \end{bmatrix} dB_t$$

15. Show that the solution $u(t,x)$ of the PDE $u_{.t} = 1/2 u_{.xx}$, i.e.,

$$\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$$

for $t > 0$ with initial condition $u(0,x) = f(x)$ can be expressed as

$$u(t,x) = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{\infty} f(y) \exp\left(-\frac{(x-y)^2}{2t}\right) dy$$

16. Consider a bounded measurable Lipschitz-function $b = b(x)$ on $\mathbb{R}$. We know that there exists a unique $X_t = X_t^x$ such that

$$dX_t = b(X_t)dt + dW_t$$

and $X_0 = x$. Use Girsanov's theorem to show that for all $K < \infty$ and all $x \in \mathbb{R}$ we have

$$P[X_t^x \geq K] > 0$$

Hint: there is no need to compute this probability.

Solutions are fairly short.