

# STAT/ELEC 331 HW 11

Problems in addition to those from the book

**Hint on Ch 7, Prob 13** Express the  $n + 1$  step transition probabilities in terms of the  $n$  step transition probabilities and the (one step) transition probabilities. Let  $n \rightarrow \infty$ . Verify the expression immediately below Definition 7.2.5 on page 418.

1. In applications, the initial value  $X_0$  of a stochastic process is typically provided, but it is also valid to view it as a random quantity. Suppose that the initial observation  $X_0$  in a Markov chain has distribution  $\mathbf{q}_0$  (viewed as a row vector). Use the law of total probability and properties of transition matrices to verify that the distribution of  $X_n$  is  $q_0 P^{(n)}$ . What happens if  $q_0$  is a stationary distribution?
2. Consider Problem 4, Chapter 7, in the book.
  - a. Describe the sequence of moves as a Markov chain by drawing a transition graph, and write down the transition probability matrix  $P$ .
  - b. Using a computer, determine the limit (numerically) of  $P^{(n)}$  as  $n \rightarrow \infty$ . Does a limit distribution exist? If so, what is it, to four digits of accuracy?
  - c. Unfortunately, this numerical approach doesn't give us exact expression for the limiting distribution. However, by problem 13 in the book, limiting distributions are stationary distributions. Perhaps we can find a stationary distribution and it will coincide with the observed limit. Find a stationary distribution by solving a linear system of equations. Don't forget the equation  $\sum_{i=1}^3 \pi_i = 1$ . Does the result agree with b?

To rigorously conclude that the stationary distribution is indeed the limit distribution, we need to verify that the Markov chain is irreducible, positive recurrent, and aperiodic. Recall that positive recurrence is equivalent to the existence of a stationary distribution, which we just established. Thus it remains to verify irreducibility and aperiodicity. Verify these properties for yourself.

### 3. Random walk on a chessboard

Suppose a king takes a random walk on a  $3 \times 3$  chessboard. Recall that a king can move one step in any direction along rows, columns, or diagonals. What is the steady-state (limiting) distribution? That is, if the random walk continues for a long, long time, and then we peek in and observe the location of the king, what is the probability of it appearing on each of the 9 squares? You may assume the fact that random walks on finite, connected graphs are ergodic.

Repeat the problem where the king is replaced by a queen, a rook, and a bishop. Note that there are two kinds of bishops.