STAT/ELEC 331 HW 4

Problems in addition to those from the book

1. In a lot of 100 light bulbs, there are five bad bulbs. An inspector inspects 10 bulbs selected at random. What is the probability that she finds at least one defective bulb?

2. Data packets arrive at an internet router at a rate of 18 packets per microsecond. Assuming a Poisson distribution, what is the probability that the router receives no more than 3 packets in a given microsecond?

3. Let r be a positive integer. The identity

$$(1-w)^{-r} = \sum_{x=r}^{\infty} \binom{x-1}{r-1} w^{x-r}$$

holds when $|w| < 1$. This series is called the *negative binomial series.*

Assume $X \sim \text{neg}(r, p)$.

a. Determine the moment generating function (MGF) of X. For what values of t does your formula hold?

b. Determine the mean and variance of X.

c. (optional - 2 points extra credit) Derive the negative binomal series from the Taylor series of $(1-w)^{-r}$.

4. Let $X \sim \text{Poi}(\lambda)$.

a. Determine the moment generating function (MGF) of X. For what values of t does your formula hold?

b. Determine the mean and variance of X.

5. McDonald's is giving away two varieties of robopets in their Happy Meals: cats and dogs. Assume that the probability of receiving a Happy Meal with a cat is p. A father promises to take his daughter to the restaurant until she has *at least one dog and one cat (and then no more).* Let the random variable X be the total number of robopets she aquires.

a. Determine the probability mass function of X.

b. What is the expected number of Happy Meals the father must buy in order to uphold his promise?