1.1-6 (a)

No. Boxes:	4	5	6	7	8	9	10	11	12	13	14	15	16	19	24
Frequency:	10	19	13	8	13	7	9	5	2	4	4	2	2	1	1

(b)

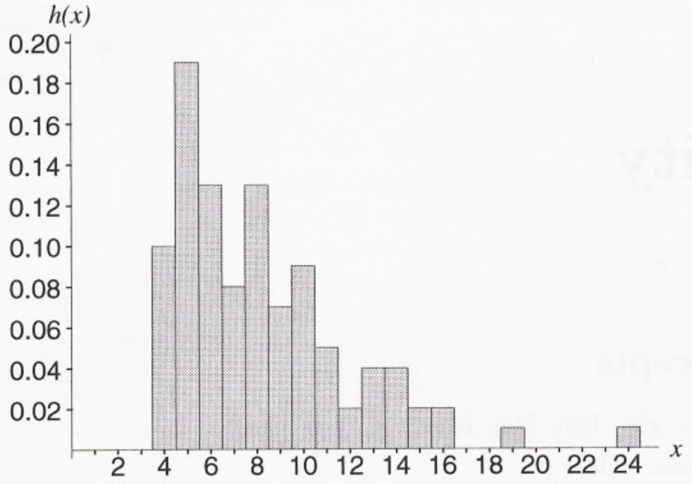

Figure 1.1–6: Number of boxes of cereal

1.1-8 (a) $f(1) = \frac{2}{10}, f(2) = \frac{3}{10}, f(3) = \frac{3}{10}, f(4) = \frac{2}{10}$.

1.1-10 This is an experiment.

1.1-12 (a) $50/204 = 0.245$; $93/329 = 0.283$;

(b) $124/355 = 0.349$; $21/58 = 0.362$;

(c) $174/559 = 0.311$; $114/387 = 0.295$;

(d) Although James' batting average is higher that Hrbek's on both grass and artificial turf, Hrbek's is higher over all. Note the different numbers of at bats on grass and artificial turf and how this affects the batting averages.

1.2 Properties of Probability

1.2-2 (a) $S =$ {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT};

(b) **(i)** 5/16, **(ii)** 0, **(iii)** 11/16, **(iv)** 4/16, **(v)** 4/16, **(vi)** 9/16, **(vii)** 4/16.

1.2-4 (a) 1/4;

(b) $P(B) = 1 - P(B') = 1 - P(A) = 3/4$;

(c) $P(A \cup B) = P(S) = 1$.

1.2-6 (a) $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

(b)
$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B) &= 0.1; \end{aligned}$$

(c) $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

1.2-8 (a)
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.7 &= 0.4 + 0.5 - P(A \cap B) \\ P(A \cap B) &= 0.2; \end{aligned}$$

(b)
$$\begin{aligned} P(A' \cup B') = P[(A \cap B)'] &= 1 - P(A \cap B) \\ &= 1 - 0.2 \\ &= 0.8. \end{aligned}$$

1.2-10
$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

1.2-12 **(a)** 1/3; **(b)** 2/3; **(c)** 0; **(d)** 1/2.

1.2-14 **(a)** $S = \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\}$;
(b) **(i)** 1/10; **(ii)** 5/10.

1.2-16 $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}.$

1.3 Methods of Enumeration

1.3-2 $(4)(3)(2) = 24.$

1.3-4 **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8.$

1.3-6 **(a)** $4\dbinom{6}{3} = 80;$

(b) $4(2^6) = 256;$

(c) $\dfrac{(4-1+3)!}{(4-1)!3!} = 20.$

1.3-8 ${}_9P_4 = \dfrac{9!}{5!} = 3024.$

1.3-10 $S =$ { FFF, FFRF, FRFF, RFFF, FFRRF, FRFRF, RFFRF, FRRFF, RFRFF, RRFFF, RRR, RRFR, RFRR, FRRR, RRFFR, RFRFR, FRRFR, RFFRR, FRFRR, FFRRR } so there are 20 possibilities.

1.3-12 $3 \cdot 3 \cdot 2^{12} = 36,864.$

1.3-14
$$\begin{aligned} \binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}. \end{aligned}$$

1.3-16
$$0 = (1-1)^n = \sum_{r=0}^{n} \binom{n}{r}(-1)^r(1)^{n-r} = \sum_{r=0}^{n} (-1)^r \binom{n}{r}.$$

$$2^n = (1+1)^n = \sum_{r=0}^{n} \binom{n}{r}(1)^r(1)^{n-r} = \sum_{r=0}^{n} \binom{n}{r}.$$

1.3.11

a) Either the National Team wins all the games or the American Team wins all 4 games.

$\boxed{2}$

b) There are 5 choose 4 ways for either team to win $\binom{5}{4}$, however you must consider the number of ways the series can end in 4 games: $2\left[\binom{5}{4} - \binom{4}{4}\right] = \boxed{8}$

c) Similar to b)

$$2\left[\binom{6}{4} - \binom{5}{4}\right] = \boxed{20}$$

d) $2\left[\binom{7}{4} - \binom{6}{4}\right] = \boxed{40}$

1.3-18
$$\binom{n}{n_1, n_2, \ldots, n_s} = \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots\binom{n-n_1-\cdots-n_{s-1}}{n_s}$$
$$= \frac{n!}{n_1!(n-n_1)!}\cdot\frac{(n-n_1)!}{n_2!(n-n_1-n_2)!}$$
$$\cdot\frac{(n-n_1-n_2)!}{n_3!(n-n_1-n_2-n_3)!}\cdots\frac{(n-n_1-n_2-\cdots-n_{s-1})!}{n_s!0!}$$
$$= \frac{n!}{n_1!n_2!\ldots n_s!}.$$

1.3-20 **(a)** $\dfrac{\binom{19}{3}\binom{52-19}{6}}{\binom{52}{9}} = \dfrac{102,486}{351,325} = 0.2917;$

(b) $\dfrac{\binom{19}{3}\binom{10}{2}\binom{7}{1}\binom{3}{0}\binom{5}{1}\binom{2}{0}\binom{6}{2}}{\binom{52}{9}} = \dfrac{7,695}{1,236,664} = 0.00622.$

1.4 Conditional Probability

1.4-2 **(a)** $\dfrac{1041}{1456};$

(b) $\dfrac{392}{633};$

(c) $\dfrac{649}{823}.$

(d) The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

1.4-4 **(a)** $P(\text{HH}) = \dfrac{13}{52}\cdot\dfrac{12}{51} = \dfrac{1}{17};$

(b) $P(\text{HC}) = \dfrac{13}{52}\cdot\dfrac{13}{51} = \dfrac{13}{204};$

(c) P(Non-Ace Heart, Ace) + P(Ace of Hearts, Non-Heart Ace)

$$= \frac{12}{52}\cdot\frac{4}{51} + \frac{1}{52}\cdot\frac{3}{51} = \frac{51}{52\cdot 51} = \frac{1}{52}.$$

1.4-6 Let $A = \{3 \text{ or } 4 \text{ kings}\}$, $B = \{2, 3, \text{ or } 4 \text{ kings}\}$.

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{N(A)}{N(B)}$$
$$= \frac{\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}{\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}} = 0.170.$$

1.4-8 **(a)** $\dfrac{8}{14}\cdot\dfrac{7}{13} = \dfrac{56}{182};$