Stat310 HW10 solutions (100'):

(5')

6.8–4 $n = \dfrac{(1.96)^2(34.9)}{(0.5)^2} = 537$, rounded up to the nearest integer.

(5')

6.8–6 $n = \dfrac{(1.96)^2(33.7)^2}{5^2} = 175$, rounded up to the nearest integer.

(10')

6.8–18 For the difference of two proportions with equal sample sizes

$$\varepsilon = z_{\alpha/2}\sqrt{\frac{p_1^*(1-p_1^*)}{n} + \frac{p_2^*(1-p_2^*)}{n}}$$

or

$$n = \frac{z_{\alpha/2}^2[p_1^*(1-p_1^*) + p_2^*(1-p_2^*)]}{\varepsilon^2}.$$

For unknown p^*,

$$n = \frac{z_{\alpha/2}^2[0.25+0.25]}{\varepsilon^2} = \frac{z_{\alpha/2}^2}{2\varepsilon^2}.$$

So $n = \dfrac{1.282^2}{2(0.05)^2} = 329$, rounded up.

(10')

7.2-1

(a) $Y \mid \theta \sim Poi(n\theta),\ \theta \sim Gamma(\alpha, \beta),\ so,\ \theta \mid y \sim Gamma(\alpha + y, \dfrac{\beta}{1+n\beta})$

(b) *Under squared loss criterion*, $w(y) = posterior\ mean = (\alpha + y)(\dfrac{\beta}{1+n\beta})$

(c) $w(y) = (\alpha + y)(\dfrac{\beta}{1+n\beta}) = (\dfrac{n}{n+1/\beta})\dfrac{y}{n} + (\dfrac{1/\beta}{n+1/\beta})\alpha\beta$

(10')

8.1–4 Using Table II in the Appendix,

(a) $\alpha = P(Y \geq 13;\ p = 0.40) = 1 - 0.8462 = 0.1538;$

(b)
$$\begin{aligned}\beta &= P(Y \leq 12;\ p = 0.60)\\ &= P(25 - Y \geq 25 - 12) \quad \text{where } 25 - Y \text{ is } b(25,\ 0.40)\\ &= 1 - 0.8462 = 0.1538.\end{aligned}$$

(10')

8.1–10 **(a)** H_0: $p = 0.14$; H_1: $p > 0.14$;

(b) $C = \{z:\ z \geq 2.326\}$ where $z = \dfrac{y/n - 0.14}{\sqrt{(0.14)(0.86)/n}}$;

(c) $z = \dfrac{104/590 - 0.14}{\sqrt{(0.14)(0.86)/590}} = 2.539 > 2.326$

so H_0 is rejected and conclude that the campaign was successful.

(10')

8.1–18 **(a)** Under H_0, $\widehat{p} = (351 + 41)/800 = 0.49$;

$$|z| = \frac{|351/605 - 41/195|}{\sqrt{(0.49)(0.51)\left(\dfrac{1}{605} + \dfrac{1}{195}\right)}} = \frac{|0.580 - 0.210|}{0.0412} = 8.99.$$

Since $8.99 > 1.96$, reject H_0.

(b)
$$\begin{aligned}0.58 - 0.21 &\pm 1.96\sqrt{\frac{(0.58)(0.42)}{605} + \frac{(0.21)(0.79)}{195}}\\ 0.37 &\pm 1.96\sqrt{0.000403 + 0.000851}\\ 0.37 &\pm 0.07 \quad \text{or} \quad [0.30, 0.44].\end{aligned}$$

It is in agreement with (a).

(c)
$$\begin{aligned}0.49 &\pm 1.96\sqrt{(0.49)(0.51)/800}\\ 0.49 &\pm 0.035 \quad \text{or} \quad [0.455, 0.525].\end{aligned}$$

(10')

8.1–20 **(a)** $z = \dfrac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{\widehat{p}(1-\widehat{p})(1/n_1 + 1/n_2)}} \ge 1.645;$

(b) $z = \dfrac{0.15 - 0.11}{\sqrt{(0.1325)(0.8675)(1/900 + 1/700)}} = 2.341 > 1.645$, reject H_0.

(c) $z = 2.341 > 2.326$, reject H_0.

(d) The p-value $\approx P(Z \ge 2.341) = 0.0096$.

(10')

8.2–6 **(a)** H_0: $\mu = 3.4$;

(b) H_1: $\mu > 3.4$;

(c) $t = (\overline{x} - 3.4)/(s/3)$;

(d) $t \ge 1.860$;

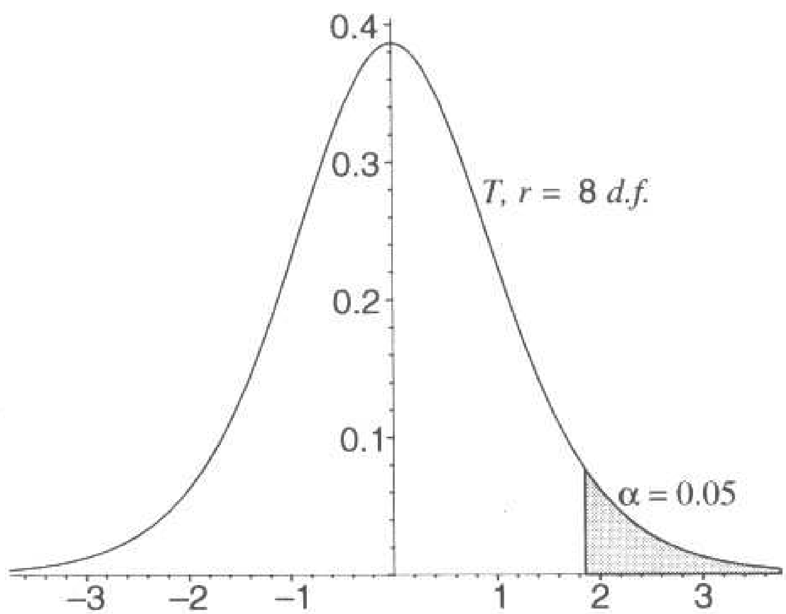

Figure 8.2–6: The critical region is $t \ge 1.860$

(e) $t = \dfrac{3.556 - 3.4}{0.167/3} = 2.802$;

(f) $2.802 > 1.860$, reject. H_0;

(g) $0.01 < p\text{-value} < 0.025$, p-value $= 0.0116$.

(10')

8.2–10 **(a)** $|t| = \dfrac{|\overline{x} - 125|}{s/\sqrt{15}} \ge t_{0.025}(14) = 2.145.$

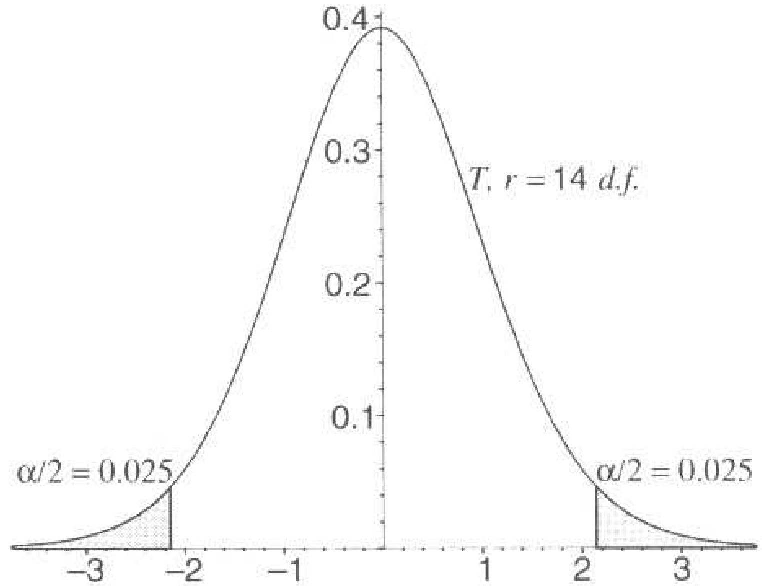

Figure 8.2–10: The critical region is $|t| \ge 2.145$

(b) $|t| = \dfrac{|127.667 - 125|}{9.597/\sqrt{15}} = 1.076 < 2.145$, do not reject H_0.

(10')

8.2–12 **(a)** The critical region is

$$\chi^2 = \frac{19s^2}{(0.095)^2} \le 10.12.$$

The observed value of the test statistic,

$$\chi^2 = \frac{19(0.065)^2}{(0.095)^2} = 8.895,$$

is less than 10.12, so the company was successful.

(b) Since $\chi^2_{0.975}(19) = 8.907$, p-value ≈ 0.025.