

# A Deep Dive Into the VIX

## Group 1

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STAT 486  
Market Models

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# Project Introduction

- Overview
- Background on the VIX
- Motivation
- Objectives



# History

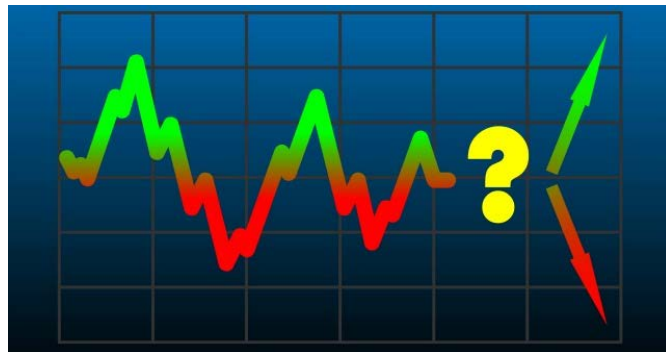
- **1993** - developed by Robert Whaley to measure 30-day volatility of S&P 100 and released by CBOE
- **2002** - moved to measure volatility of the S&P 500
- **2004, 2006** - futures and options added
- **2014** - switched from monthly to weekly options
- **2015** - average trading activity in VIX options and futures grew to nearly 800,000 contracts per day





# What is the VIX?

- Derived from options from S&P 500 tickers
- Measures the expected volatility of the market in the following 30 days
  - Does not predict direction of movement will be
- High volatility leads to increased options prices due to a higher likelihood of movement







# How The VIX Is Calculated

The formula for calculating the VIX is as follows:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[ \frac{F}{K_0} - 1 \right]^2$$

Where:

$K_i$  = Strike Price of  $i^{th}$  out-of-the-money option

$T$  = Time to Expiration

$Q(K_i)$  = The midpoint of the bid-ask spread for each option with strike  $K_i$

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

$F$  = Forward Index Level Derived From Index Option Prices

$$\sigma = VIX/100 \Rightarrow VIX = \sigma \times 100$$

$K_0$  = First Strike Below the Forward Index Level,  $F$

$R$  = Risk-Free Interest Rate to Expiration



## VIX Calculation (cont.)

$$\sigma^2 = \frac{21,600}{525,600} \times 0.066472 \times \left( \frac{61,920 - 43,200}{61,920 - 21,600} \right) +$$
$$\frac{61,920}{525,600} \times 0.063667 \times \left( \frac{43,200 - 21,600}{61,920 - 21,600} \right) \times \frac{525,600}{43,200}$$

$$\sigma^2 = 0.0643180321$$

$$\sigma = 0.253610$$

$$VIX = 100 \times \sigma = 25.36$$

# How to trade the VIX, and use the VIX to trade

- Futures Contracts
- Options Contracts
- Trade ETFs and ETNs that are tied to the VIX
- If VIX is high, options strategies that benefit from a sharp move in either direction
  - Debit iron condor, Strangles, Straddles, etc.

# VIX Alternatives

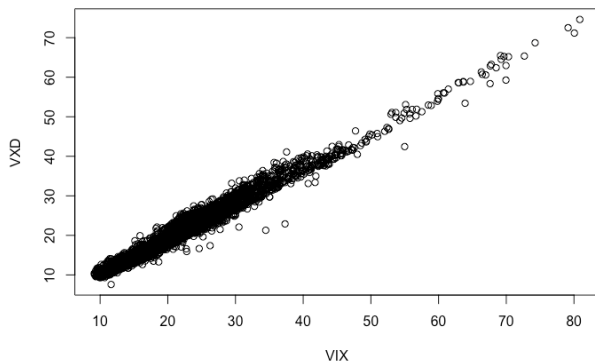
(US Only)

- VXX
- UVXY
- VXO
- VXN
- VXD

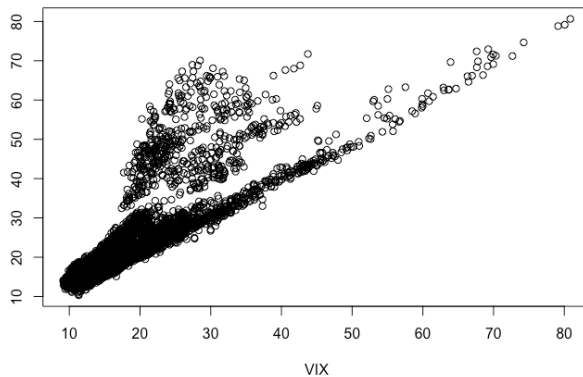


# VIX Alternatives (US Only)

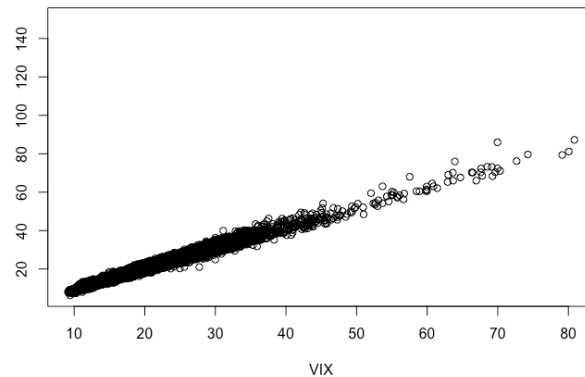
Comparison of VIX and VXD



Comparison of VIX and VXN



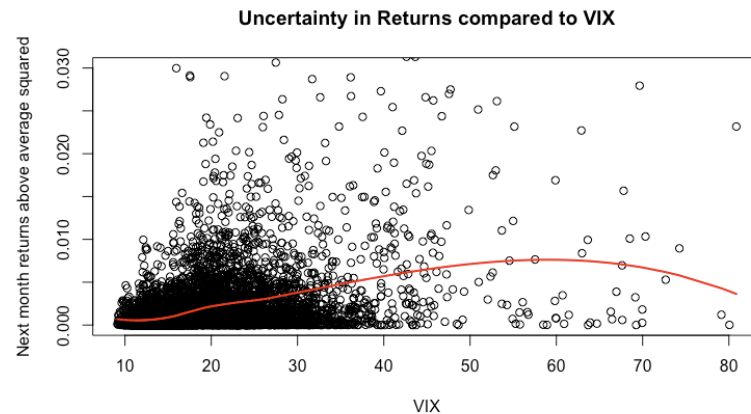
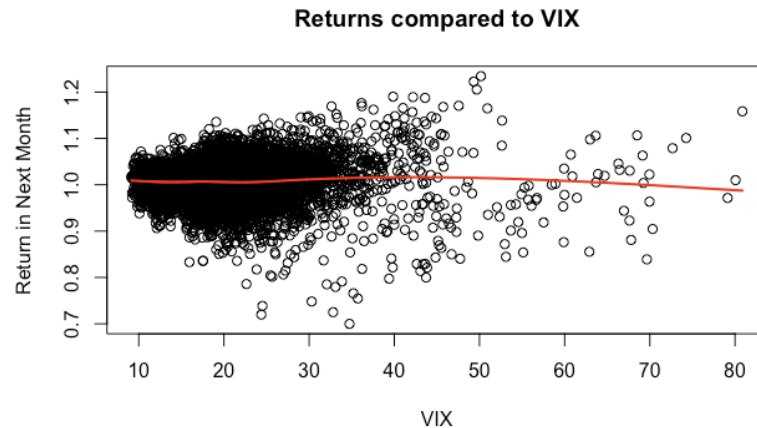
Comparison of VIX and VXO





# Determining Market Direction With VIX

Future returns and uncertainty  
compared to current VIX.





# Determining Market Direction With VIX (cont.)

Decile	Min. VIX	Max. VIX	Future Returns in the Next Time Period					Standard Deviation				
			Day	Week	Month	Quarter	Year	Day	Week	Month	Quarter	Year
1	9.14	11.92	0.07	0.19	0.75	2.29	9.52	0.47	1.09	2.2	4.21	9.28
2	11.92	13.02	0.05	0.13	0.48	2.16	10.49	0.51	1.25	2.59	4.3	8.65
3	13.02	14.26	0.02	0.03	0.52	2.15	9.94	0.6	1.53	2.55	3.91	8.76
4	14.27	15.76	0.03	0.1	0.55	1.98	10.71	0.68	1.59	2.84	4.78	9.76
5	15.76	17.26	0.03	0.14	0.52	1.55	11.47	0.79	1.71	3.5	6.23	12.15
6	17.26	19.22	0.06	0.25	0.8	0.83	7.14	0.92	1.93	4.39	8.06	18.08
7	19.22	21.25	0.07	0.19	0.34	-0.01	3.94	1.03	2.22	4.9	10.24	22.95
8	21.25	23.98	0.06	0.15	0.33	0.94	-0.73	1.12	2.53	5	9.55	22.91
9	23.99	28.38	-0.04	-0.01	0.63	2.11	0.06	1.33	2.79	5.8	8.75	23.41
10	28.4	80.86	-0.06	0.28	1.28	4.72	16.06	2.33	4.53	7.77	11.06	18.73

Took deciles and found the geometric mean and standard deviations of future returns over different time periods



# VIX Convergence vs. Divergence

	Quintile	Smoothed Lagged		Future Returns in the Next Time Period					Standard Deviation				
		Change in VIX	Returns	Day	Week	Month	Quarter	Year	Day	Week	Month	Quarter	Year
Bear Convergence	1	0.04	-0.03	-0.02	0.19	0.52	1.54	6.09	1.06	2.11	4.09	7.53	19.07
	2	0.08	-0.06	0.04	0.25	1.19	1.96	7.17	1.09	2.16	4.48	7.40	18.60
	3	0.11	-0.11	0.01	0.32	1.41	2.60	5.89	1.35	3.01	5.58	9.10	21.78
	4	0.19	-0.17	0.16	0.14	0.72	2.14	3.75	1.51	3.09	6.45	9.72	23.87
	5	0.44	-0.34	0.12	0.61	0.95	2.30	9.29	2.53	4.84	8.36	12.02	22.14
Bear Divergence	1	-0.03	-0.02	0.02	-0.09	0.54	0.59	3.52	0.99	2.45	4.46	8.49	18.86
	2	-0.04	-0.04	0.04	-0.25	0.85	0.84	1.68	1.12	2.42	4.13	8.86	21.13
	3	-0.06	-0.06	0.05	0.36	0.41	-0.37	2.58	1.32	2.65	5.05	11.75	22.57
	4	-0.08	-0.07	0.40	0.47	0.86	-0.95	-4.57	1.35	2.67	5.81	13.08	28.46
	5	-0.16	-0.13	-0.06	0.02	-0.04	2.82	11.66	1.86	4.07	8.65	12.86	26.69
Bull Convergence	1	-0.03	0.05	0.06	0.16	0.34	1.59	8.31	0.72	1.77	3.35	6.35	13.02
	2	-0.05	0.08	0.00	0.02	0.32	1.68	8.68	0.68	1.58	3.08	6.00	12.47
	3	-0.07	0.11	0.00	0.01	0.47	1.87	8.73	0.70	1.59	3.14	6.31	13.69
	4	-0.11	0.14	0.00	0.02	0.18	1.38	5.94	0.83	1.72	3.73	6.80	17.00
	5	-0.19	0.21	0.03	0.21	1.01	2.14	8.90	0.99	2.08	4.27	7.68	17.37
Bull Divergence	1	0.02	0.04	-0.09	0.17	0.70	2.48	9.12	0.90	1.68	3.08	5.50	12.17
	2	0.04	0.05	0.02	0.31	0.82	2.57	8.52	0.81	1.57	3.47	5.08	12.33
	3	0.05	0.07	0.02	0.16	0.70	2.52	11.38	0.85	1.92	3.65	5.45	12.34
	4	0.06	0.10	0.05	0.06	1.10	2.71	12.81	0.85	2.10	3.69	5.24	13.16
	5	0.11	0.17	-0.09	-0.13	-0.11	2.29	11.37	1.07	2.38	4.08	5.87	16.15

Bull and Bear Markets with VIX converging and diverging. Split into quintiles and then measured future returns and uncertainty.



# VIX vs. Market Sentiment

Market Sentiment includes:

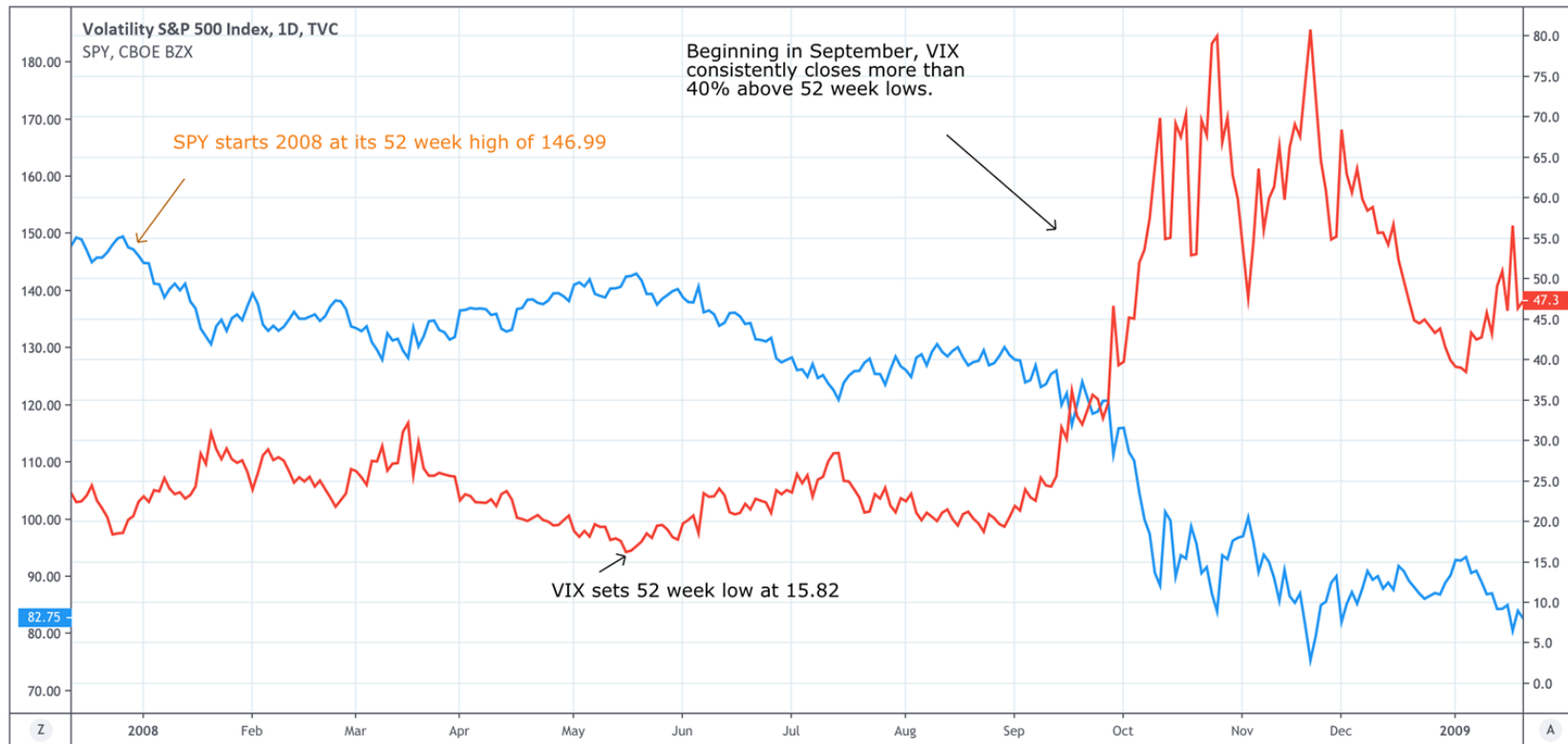
- VIX (and friends)
- High-low index
- Moving Averages
- Put-call ratios
- Fear-greed index

TVC:VIX, 1D 38.2 ▼ -2.0 (-4.89%) O:39.5 H:40.3 L:37.6 C:38.2



2007 pre GFC

TVC:VIX, 1D 38.2 ▼ -2.0 (-4.89%) O:39.5 H:40.3 L:37.6 C:38.2



TVC:VIX, 1D 38.2 ▼ -2.0 (-4.89%) O:39.5 H:40.3 L:37.6 C:38.2

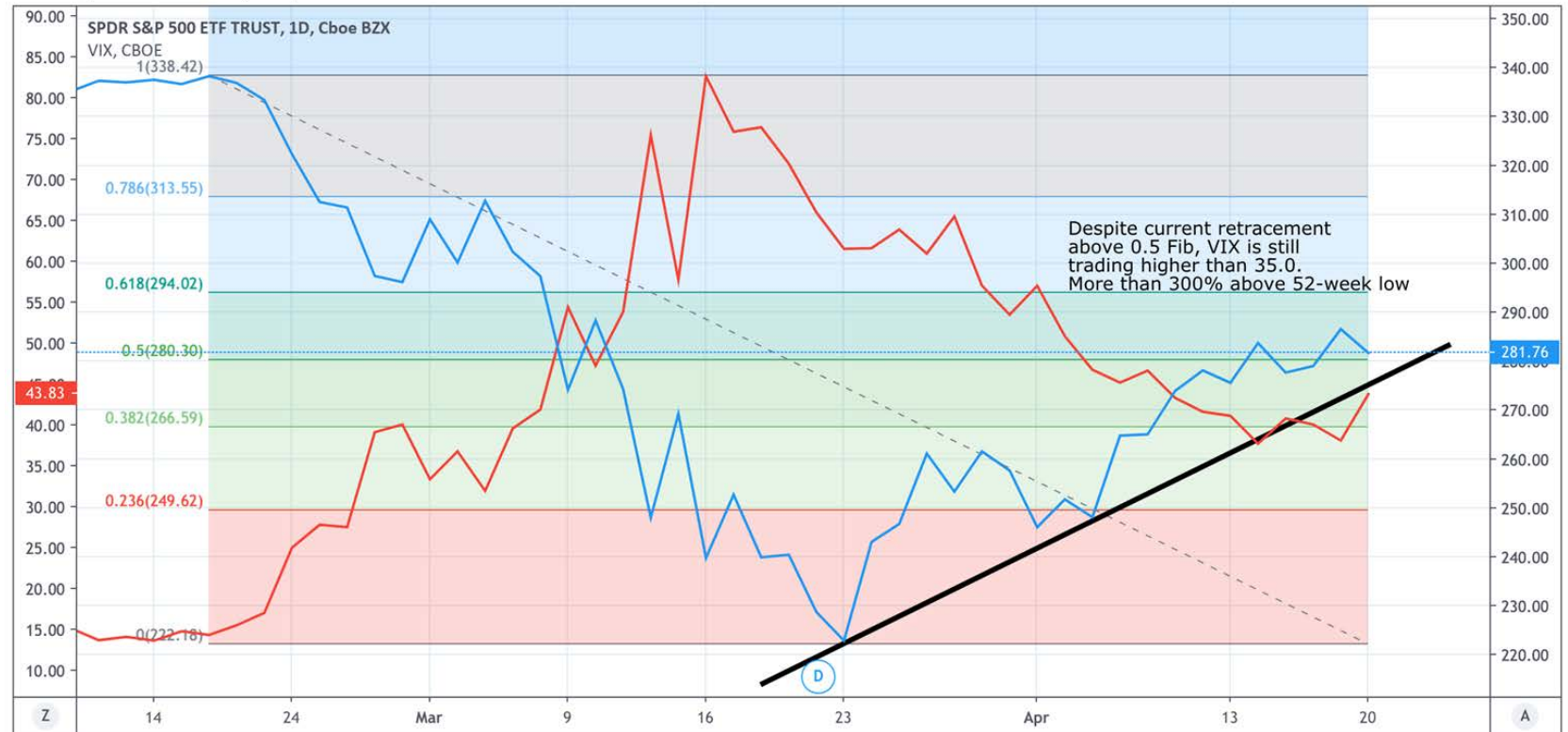


BATS:SPY, 1D 281.76 ▼ -4.88 (-1.7%) O:282.61 H:286.79 L:281.35 C:281.76



2008 fib

BATS:SPY, 1D 281.76 ▼ -4.88 (-1.7%) O: 282.61 H: 286.79 L: 281.35 C: 281.76



Current (cont)



# Conclusion: What is the VIX?



- Volatility gauge for options market, calculated by CBOE
- Marker of uncertainty for near term markets
- Interesting tool for analysis of recessions and booms

# References

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"VIX Calculation Explained." *Macroption*, [www.macroption.com/vix-calculation/](http://www.macroption.com/vix-calculation/)



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## Raw Data

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Wharton Research Data Services. "CRSP Index File on the S&P 500" wrds.wharton.upenn.edu, accessed 04/16/2020.

## Charts

"Free Stock Charts, Stock Quotes and Trade Ideas." *TradingView*, [www.tradingview.com/](https://www.tradingview.com/).

# Max-Median TO THE MAX

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Aneel Damaraju

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SUMMARY OF MAX-MEDIAN RULE



OPTIMIZING MAX-MEDIAN RULE BY PORTFOLIO SIZE



MEASURING MAX-MEDIAN RETURNS BY SECTOR



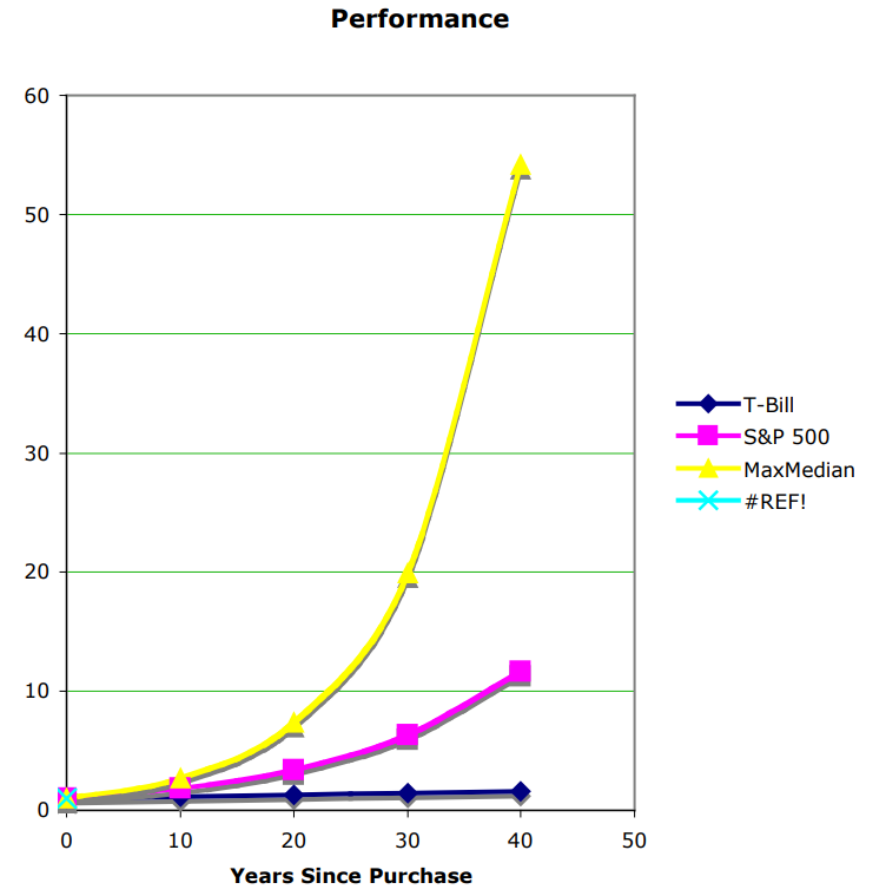
EVALUATING MAX-MEDIAN RETURN ON THE DOW



COMPARING DIFFERENT TECHNIQUES THROUGH THE SHARPE RATIO

# What is the Max-Median Rule?

- Simple algorithmic investing strategy
- Can be performed by any individual investor
- Has been shown to historically beat the S&P index by about 50%



Baggett

## Basic Max-Median Methodology



1. Select the top 20 stocks from year  $t-1$  that had the highest daily median returns.
2. Invest in these stocks equally at the beginning of year  $t$
3. Hold the stocks throughout the duration of year  $t$
4. Sell the stocks at the end of year  $t$
5. Repeat these steps for year  $t+1$

## Optimizing By Portfolio Size

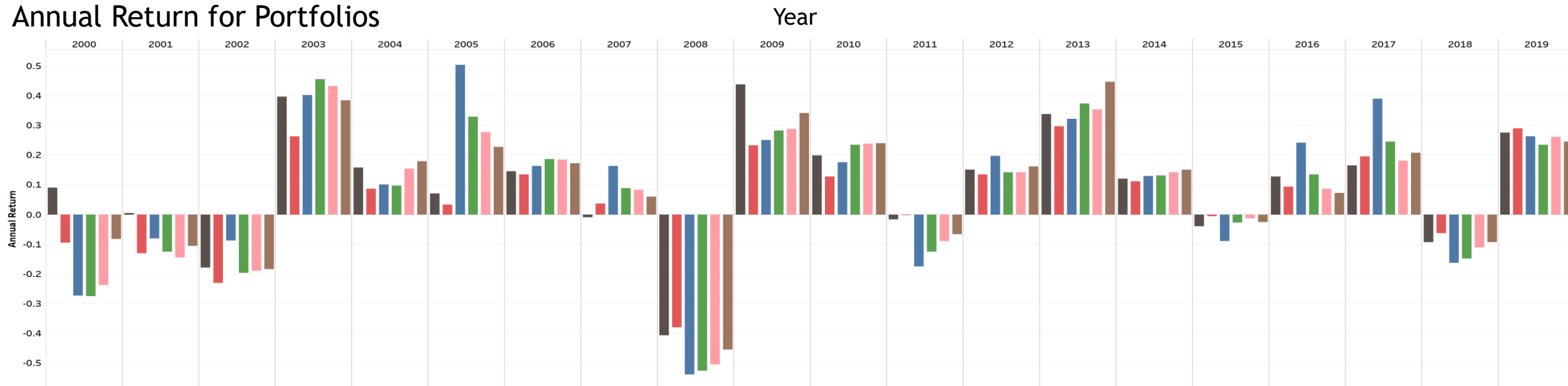
- The default Max-Median portfolio size is  $n=20$
- Different sizes were tested at  $n=10, 30$ , &  $50$
- Yearly and cumulative returns were calculated for these portfolio sizes



# Optimizing By Portfolio Size



Annual Return for Portfolios



## Measure Names

- S&P 500 Equal Weighted w/o Dividends
- S&P 500 Value Weighted w/o Dividends
- Top 10 Max-Med
- Top 20 Max-Med
- Top 30 Max-Med
- Top 50 Max-Med

“Top 50 Max-Med saw the highest cumulative return”

# Optimizing By Portfolio Size



Index	Cumulative Return
S&P Market-Weighted	123%
S&P Equal-Weighted	<b>347%</b>
S&P Top 20 Max-Med	135%
S&P Top 10 Max-Med	216%
S&P Top 30 Max-Med	158%
S&P Top 50 Max-Med	<b>288%</b>



# Measuring Returns by Sector

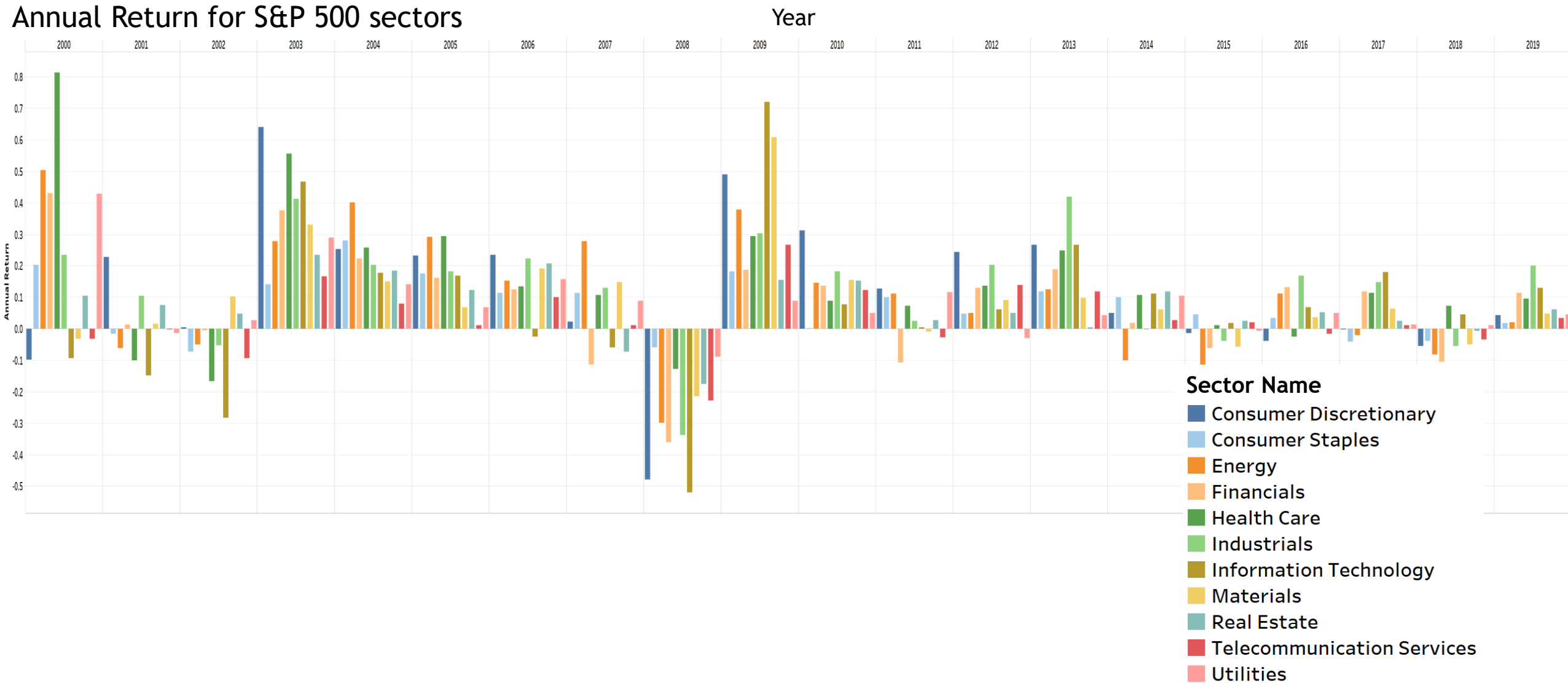


- In 1999, the S&P 500 began dividing its stocks into sectors. These are the sectors that the index currently uses and the number of stocks in each:
  - Communication Services (26)
  - Consumer Discretionary (62)
  - Consumer Staples (33)
  - Energy (27)
  - Financials (66)
  - Health Care (60)
  - Industrials (72)
  - Materials (28)
  - Real Estate (31)
  - Technology (71)
  - Utilities (28)
- We applied the Max-Median rule to each sector and calculated yearly and cumulative returns

# Measuring Returns by Sector



Annual Return for S&P 500 sectors



# Measuring Returns by Sector



S&P 500 Sector	Cumulative Return
Consumer Discretionary	533%
Consumer Staples	279%
Energy	436%
Financials	255%
<b>Health Care</b>	<b>1066%</b>
Industrials	843%
Information Technology	111%
Materials	369%
Real Estate	259%
Telecommunication Services	79%
Utilities	323%

# Max-Median on the Dow

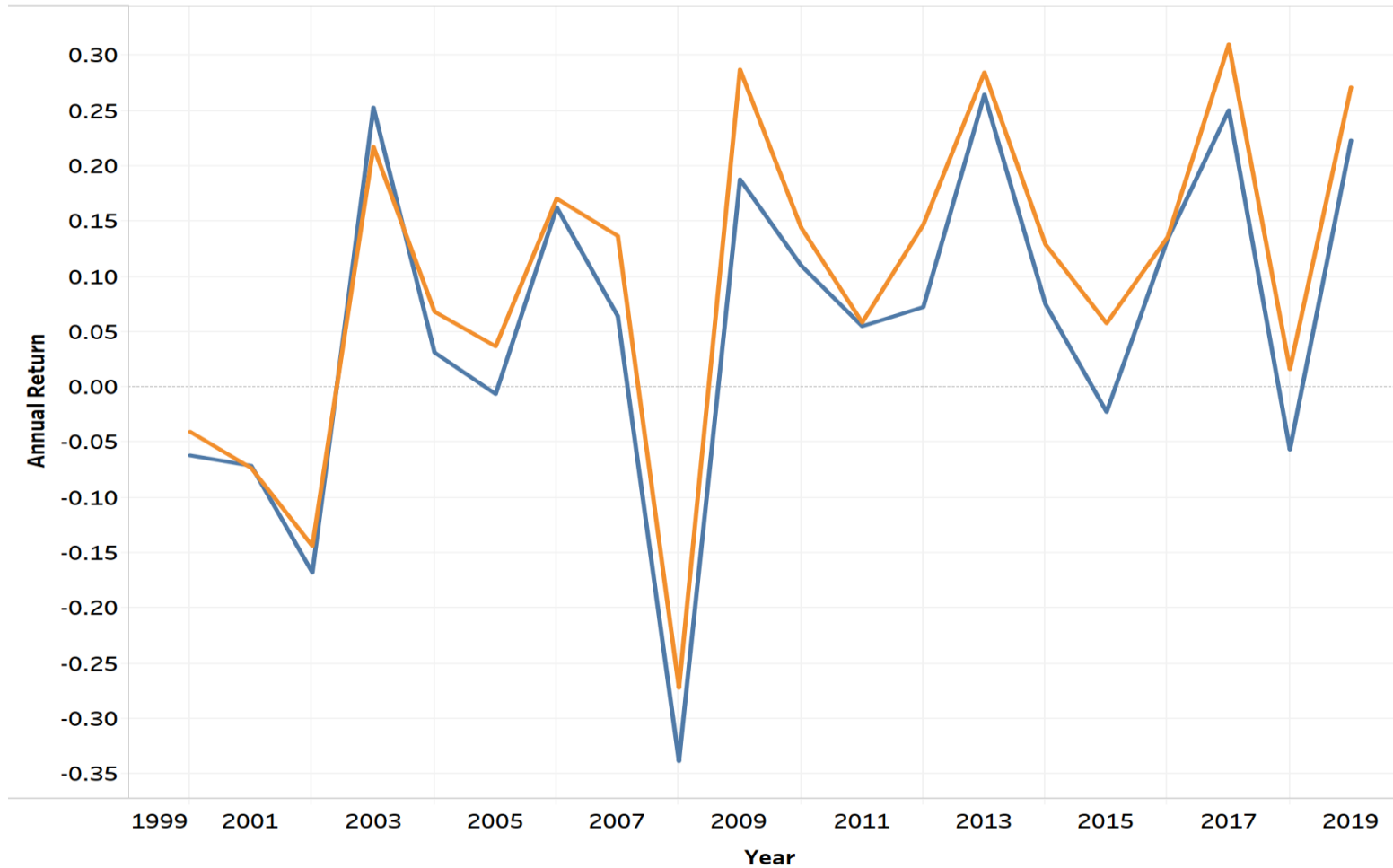


- In order to see whether the Max-Median Rule could be applied to indices other than the S&P, we applied it to the Dow Jones Industrial Average.
  - $n = 20$
  - Did not vary  $n$  because the Dow only has 30 stocks
  - Yearly and cumulative returns were calculated

# Max-Median on the Dow



DJIA Annual Returns



Investment Strategy

- DJIA
- DJIA Max-Median

# Max-Median on the Dow



Index	Cumulative Return
DJIA	148%
DJIA Max-Median	427%

## Comparing Via the Sharpe Ratio

- The Sharpe Ratio is a measurement of return compared to risk =  $(R_p - R_f)/\sigma_p$ 
  - $R_p$  is the return on the portfolio
  - $R_f$  is the risk-free rate
  - $\sigma_p$  is the standard deviation of the portfolio's excess return
- For each variation of the Max-Median Rule, we calculated the Sharpe Ratio to illustrate the tradeoff between risk and return.



# Sharpe Ratio Results



Portfolio	Average Excess Return	Standard Deviation	Sharpe Ratio
DJIA Max-Med	8.08%	15.26%	0.53
DJIA	4.17%	15.41%	0.27
Consumer Discretionary	10.69%	23.45%	0.46
Consumer Staples	5.65%	9.19%	0.61
Energy	8.93%	19.32%	0.46
Financials	6.42%	17.24%	0.37
Health Care	13.32%	21.60%	0.62
<b>Industrials</b>	<b>11.66%</b>	<b>16.96%</b>	<b>0.69</b>
Information Technology	5.27%	25.49%	0.21
Materials	7.47%	16.39%	0.46
Real Estate	5.39%	9.46%	0.57
Telecommunication Services	1.82%	10.71%	0.17
Utilities	6.36%	10.48%	0.61
S&P 500 Value Weighted w/o Dividends	3.99%	17.61%	0.23
S&P 500 Equal Weighted w/o Dividends	8.02%	19.63%	0.41
Top 20 Max-Med	5.92%	24.39%	0.24
Top 10 Max-Med	7.80%	25.60%	0.30
Top 30 Max-Med	6.03%	23.02%	0.26
Top 50 Max-Med	7.72%	21.60%	0.36



**Thank You!**  
**Questions?**

# References

- Baggett(2009). “Everyman’s MaxMedian Rule for Portfolio Management”.



# Appendix

# Table 1: Max-Median Returns by Portfolio Size

Year	S&P 500 Value Weighted w/o Dividends	S&P 500 Equal Weighted w/o Dividends	Top 20 Max-Med	Top 10 Max-Med	Top 30 Max-Med	Top 50 Max-Med
2000	-9.55%	8.97%	-27.48%	-27.39%	-23.84%	-8.27%
2001	-13.07%	0.54%	-12.58%	-8.11%	-14.48%	-10.65%
2002	-23.06%	-17.86%	-19.68%	-8.86%	-18.97%	-18.39%
2003	26.39%	39.76%	45.48%	40.23%	43.29%	38.51%
2004	8.74%	15.70%	9.76%	10.00%	15.40%	17.91%
2005	3.29%	7.02%	32.96%	50.32%	27.78%	22.76%
2006	13.51%	14.59%	18.69%	16.24%	18.43%	17.15%
2007	3.73%	-0.89%	8.92%	16.36%	8.35%	5.92%
2008	-37.96%	-40.68%	-52.62%	-53.86%	-50.53%	-45.56%
2009	23.29%	43.80%	28.27%	25.05%	28.84%	34.11%
2010	12.70%	19.82%	23.47%	17.51%	23.85%	24.07%
2011	-0.28%	-1.72%	-12.50%	-17.49%	-9.05%	-6.60%
2012	13.41%	15.08%	14.16%	19.68%	14.22%	16.14%
2013	29.72%	33.80%	37.42%	32.19%	35.40%	44.69%
2014	11.21%	12.12%	13.03%	12.98%	14.27%	14.99%
2015	-0.67%	-4.08%	-2.71%	-9.04%	-1.26%	-2.51%
2016	9.33%	12.76%	13.54%	24.15%	8.70%	7.17%
2017	19.53%	16.45%	24.48%	39.03%	18.13%	20.74%
2018	-6.35%	-9.34%	-14.81%	-16.27%	-11.08%	-9.28%
2019	28.90%	27.58%	23.47%	26.23%	26.07%	24.53%

Table 2: Max-Median Returns by Sector

Year	Consumer Discretionary	Consumer Staples	Energy	Financials	Health Care	Industrials	Information Technology	Materials	Real Estate	Telecommunications Services	Utilities
2000	-9.87%	20.26%	50.36%	43.07%	81.40%	23.42%	-9.20%	-3.04%	10.60%	-3.05%	42.96%
2001	22.77%	-1.55%	-6.19%	1.32%	-10.09%	10.53%	-14.68%	1.68%	7.67%	0.05%	-1.23%
2002	0.49%	-7.31%	-5.03%	-0.50%	-16.61%	-5.16%	-28.20%	10.30%	4.90%	-9.35%	2.83%
2003	64.07%	14.13%	27.82%	37.75%	55.57%	41.31%	46.77%	33.07%	23.61%	16.74%	29.10%
2004	25.28%	28.15%	40.18%	22.43%	25.86%	20.25%	17.78%	15.12%	18.48%	8.07%	14.12%
2005	23.33%	17.63%	29.23%	16.28%	29.48%	18.25%	17.00%	6.91%	12.27%	1.19%	7.00%
2006	23.49%	11.54%	15.39%	12.63%	13.52%	22.31%	-2.51%	19.22%	20.77%	10.10%	15.85%
2007	2.38%	11.44%	27.83%	-11.35%	10.65%	13.00%	-5.88%	14.78%	-7.24%	1.30%	8.98%
2008	-47.94%	-5.87%	-29.77%	-35.95%	-12.82%	-33.64%	-51.89%	-21.38%	-17.47%	-22.73%	-8.94%
2009	48.98%	18.22%	37.89%	18.79%	29.36%	30.34%	72.10%	60.88%	15.49%	26.79%	8.93%
2010	31.37%	0.34%	14.73%	13.62%	9.04%	18.17%	7.88%	15.48%	15.43%	12.38%	5.16%
2011	12.77%	10.12%	11.28%	-10.57%	7.42%	2.55%	0.60%	-0.77%	2.68%	-2.71%	11.57%
2012	24.48%	4.89%	4.95%	13.12%	13.63%	20.26%	6.23%	9.25%	4.97%	13.87%	-2.85%
2013	26.80%	11.82%	12.62%	19.02%	24.95%	41.99%	26.74%	9.90%	0.58%	11.90%	4.38%
2014	4.95%	10.11%	-10.02%	1.82%	10.76%	-0.06%	11.32%	6.30%	11.97%	2.89%	10.61%
2015	-1.43%	4.56%	-12.61%	-6.04%	1.28%	-3.80%	1.85%	-5.62%	2.63%	2.00%	-0.73%
2016	-3.72%	3.45%	11.20%	13.20%	-2.41%	16.84%	6.88%	3.75%	5.38%	-1.46%	5.14%
2017	-0.18%	-4.06%	-2.01%	11.81%	11.52%	14.89%	17.96%	6.47%	2.49%	1.19%	1.45%
2018	-5.47%	-3.81%	-8.21%	-10.41%	7.27%	-5.46%	4.54%	-4.87%	-0.60%	-3.27%	1.28%
2019	4.27%	1.81%	2.01%	11.34%	9.61%	20.19%	13.10%	4.94%	6.09%	3.41%	4.50%

# Table 3: Max-Median Returns for DJIA

Year	DJIA	DJIA Max-Med
2000	-6.17%	-4.03%
2001	-7.10%	-7.31%
2002	-16.76%	-14.36%
2003	25.32%	21.77%
2004	3.15%	6.84%
2005	-0.61%	3.70%
2006	16.29%	17.08%
2007	6.43%	13.70%
2008	-33.84%	-27.19%
2009	18.82%	28.76%
2010	11.02%	14.46%
2011	5.53%	5.86%
2012	7.26%	14.72%
2013	26.50%	28.50%
2014	7.52%	12.93%
2015	-2.23%	5.79%
2016	13.42%	13.60%
2017	25.08%	31.04%
2018	-5.63%	1.64%
2019	22.34%	27.15%

# **Volume-Based Trading Strategies**

Team 3

Autumn Engebretson

Eduardo Camou

Eli Smith

Maher Nammour

## **Agenda**

- Volume
- Volume-based trading strategies
- Our Strategy
- Results
- Q&A



# **Volume**

- Volume describes the number of shares of a security traded during a given period of time.

- It's an important indicator in technical analysis because it is used to measure the relative significance of a market move.
- The higher the volume during a price move, the more significant the move and the lower the volume during a price move, the less significant the move.

# **Volume-based trading strategies**

## I. Breakouts and Volume

- Traders will look for breaks of support and resistance to enter positions. There are two key components to confirm a breakout: price and volume.
- When stocks break critical levels without volume, you should consider the breakout suspect and prime for a reversal off the highs/lows.
- On the breakout, volume should pick up.



Triangle Breakout With High Volume. Investopedia

## II. Trading Stocks and Volume

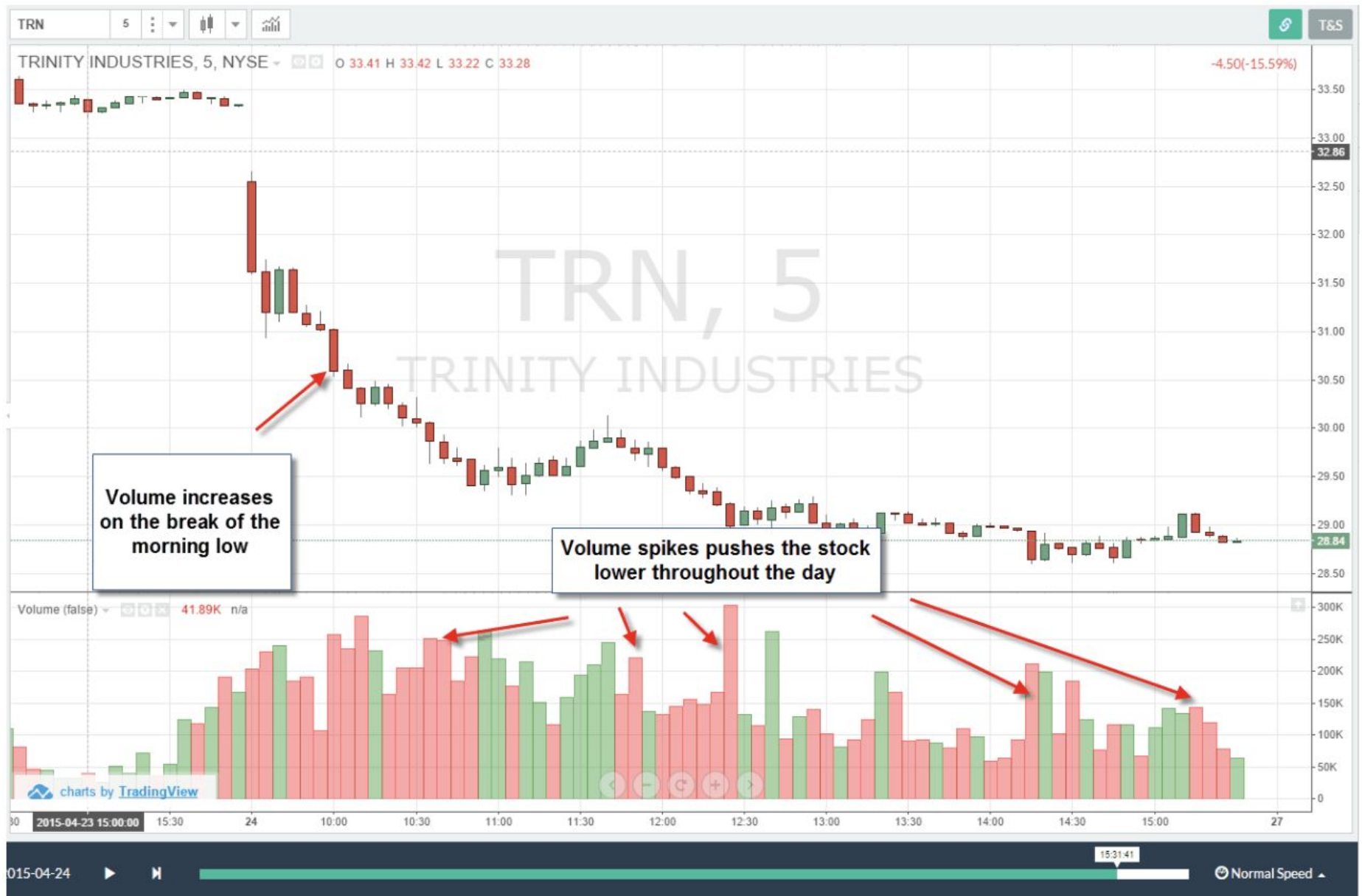
- When a stock is moving higher in a stair-step approach, a volume increase on each successive high or decrease on each pullback is used to confirm the health of the trend.
- This sort of confirmation in the volume activity is usually a result of a stock in an impulsive phase of a trend.

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## III. Volume Spikes

- Volume spikes are usually the result of news-driven events. They are defined as a daily volume that is two standard deviations away from the recent average.
- Better to be a buyer than a seller.
- Present an opportunity for a counter move position (take a short).

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## IV. Trading the Failed Breakout

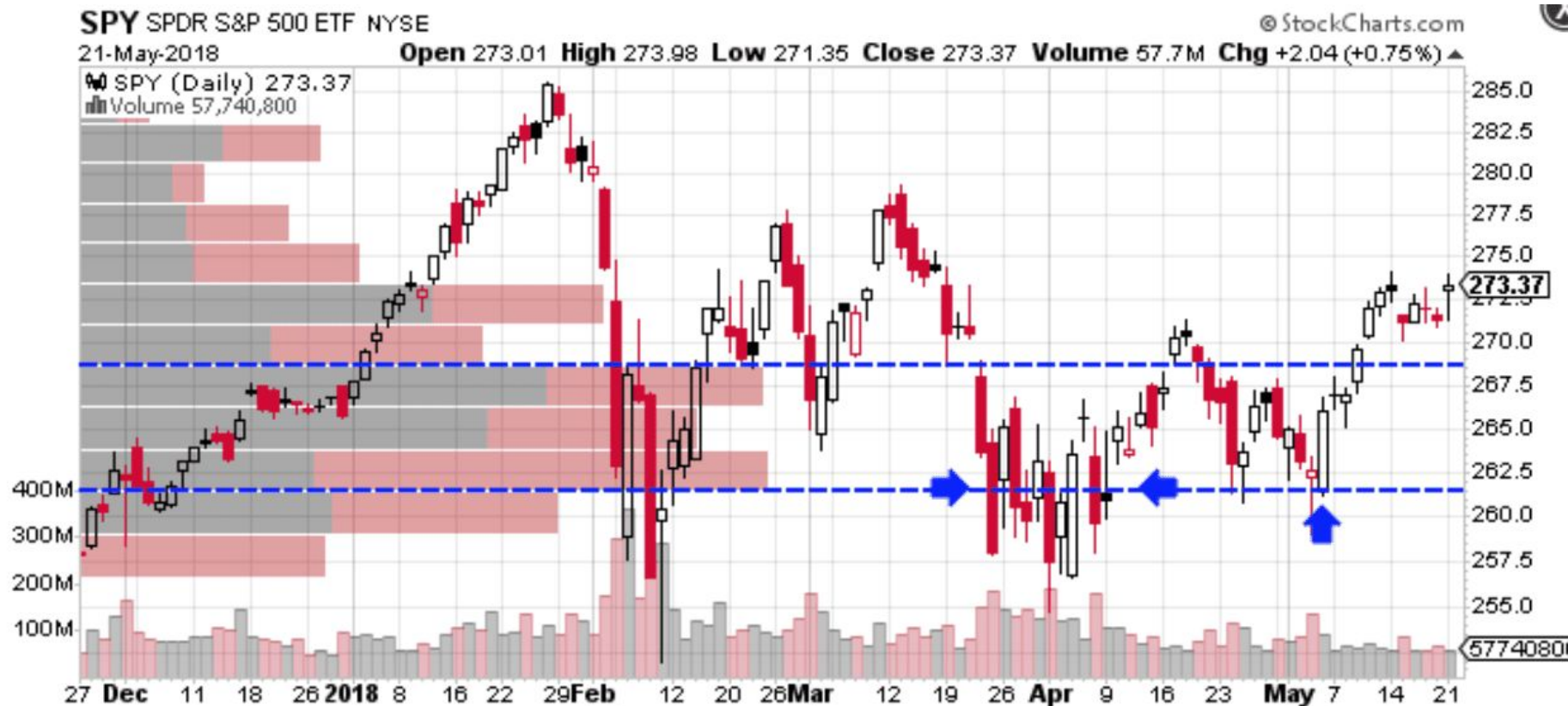
- Volume is a good indicator that a trade is failing.
- If volume dries up on the breakout, traders try to get out within a few candles. The pick of the first candlestick is a logical point to exit the trade.



## V. Overlay of Volume on Price

- Plot volume on the x-axis and price on the y-axis, allowing the trader to see where the concentration of orders took place.
- This can provide the trader with a clear view into where there are many traders and they can then use this to validate a particular support or resistance level.

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## **Our strategy**



## Inverse the Market on Low-Volume Days

If Today's Volume is:	*If Today Went Up*:	*If Today Went Down*:	If Today Ended Even:
*Far Below Average*	Sell	Buy	*Maintain Position*
Near Average	Buy	*Sell*	Maintain Position
*Far Above Average*	Buy	Sell	Maintain Position

Monday	Tuesday	Wednesday	Thursday	Friday
<p>Volume: Average SPY: Up Strategy says to Buy</p>	<p>Volume: Low SPY: Up Strategy says to Sell</p>	<p>Volume: High SPY: Down Strategy says to Sell</p>	<p>Volume: Average SPY: Even Strategy says to Maintain</p>	<p>Volume: Low SPY: Down Strategy says to Buy</p>

## Flexibility in the Strategy

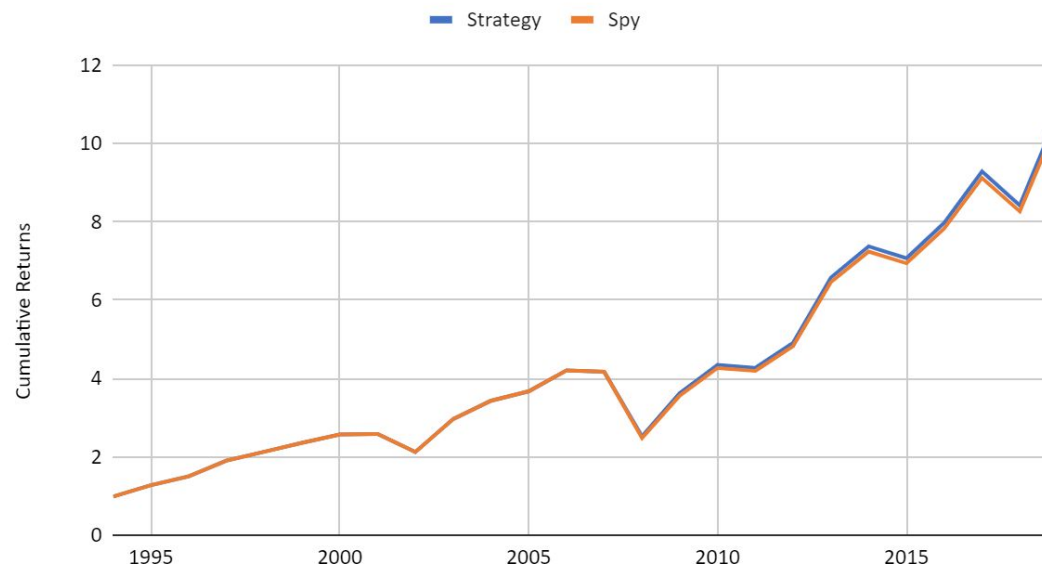
- What does High/Low Volume mean?
  - Or for prices to Rise or Fall?
- Triple 2 Rule:
  - If we are out of the market, and the strategy says “Maintain” for (3) days in a row, go ahead and buy back in
- Rolling Vs Dumb Average:
  - Rolling: Average of last 250 trading days’ volume
  - Dumb: Simply use last year’s average
- Re-buy in at start of each year
  - Even if we are out of market LTDOY, still re-buy in FTDOY

## **Results**

## Results from \$SPY ETF

- Best Strategy's Parameters:
  - Gain/Loss Movement: 9% Strategy CAGR: 1.0955
  - High/Low Volume: 30% SPY CAGR: 1.0948
  - Number of Trades: 2 (Sell in 2008, then rebuy 2 weeks later)

Volume Strategy vs Buy and Hold on SPY ETF

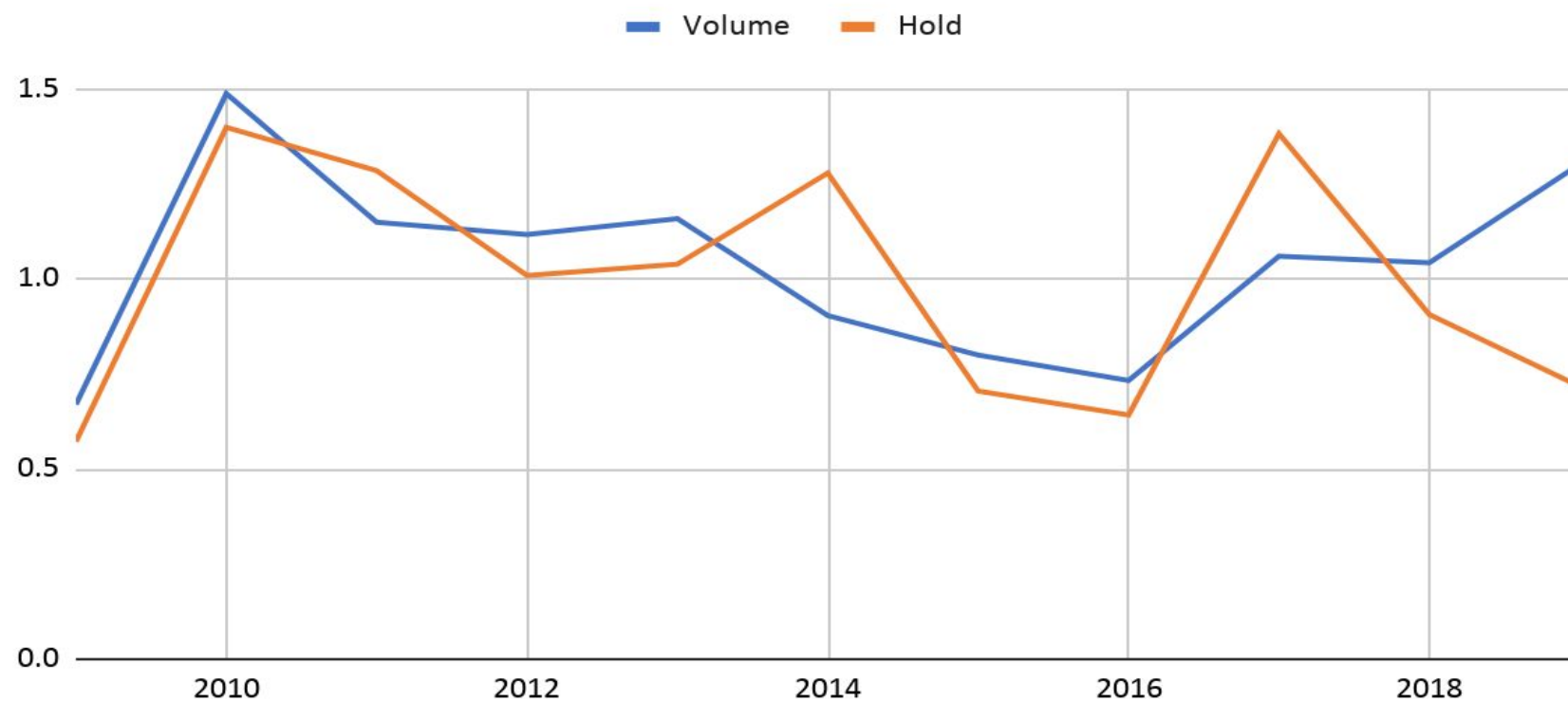


## Application to Other Securities

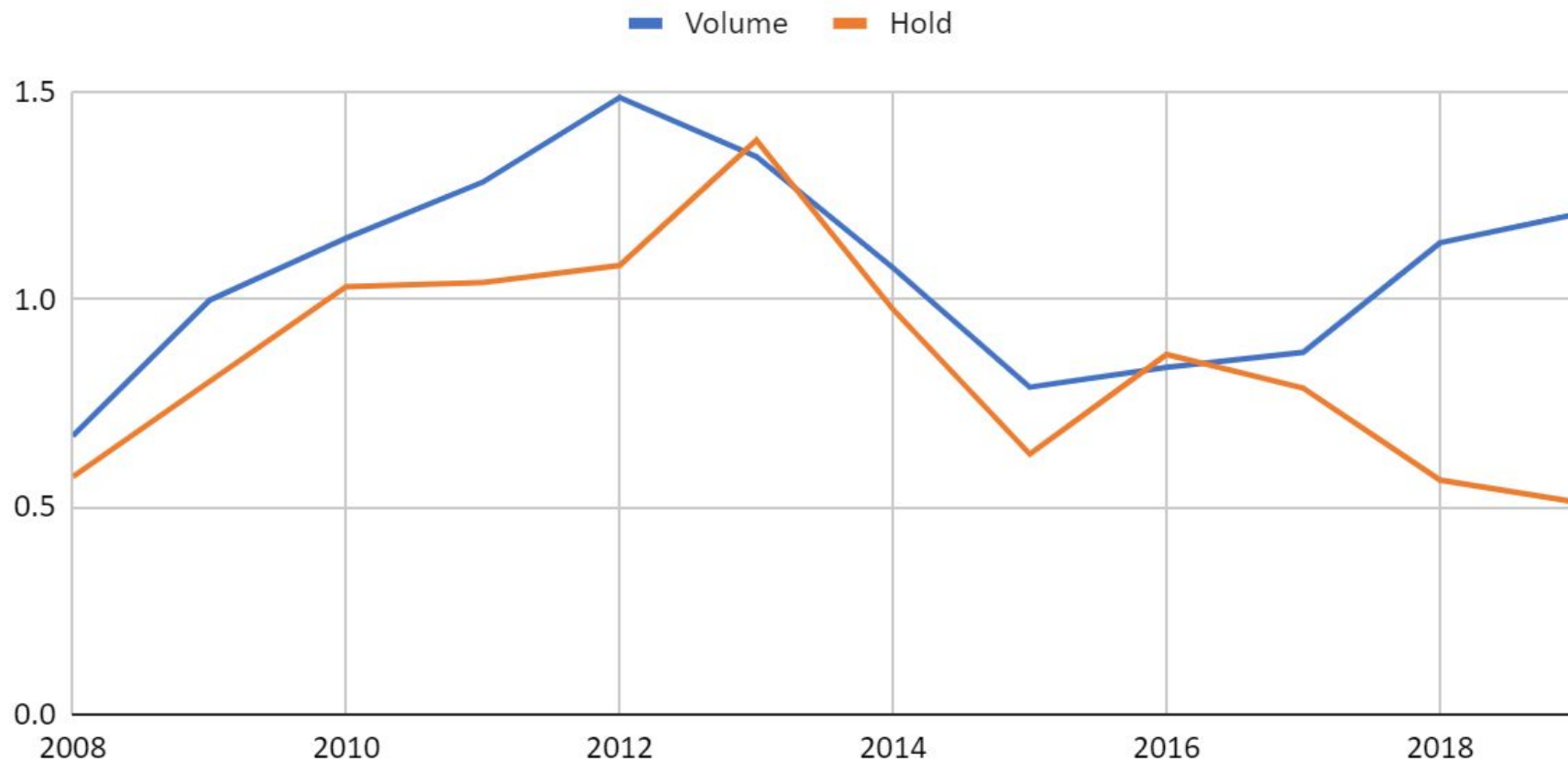
- Oil is more volatile: \$XOP ETF from 2008 to 2019
  - Buy and Hold: -51.18%
  - Strategy: 20.44% (491 Trades)
    - More profits could be made by shorting when strategy says to
- Apple is famously profitable: \$AAPL from 1984 to 2019
  - Buy and Hold: 84,860%
  - Strategy: 505,178.6% (26 Trades)
    - Spread out trades in 1987-2001, then no trades until 2008
- Testing the 2008 Crash: BAC from 2000 to 2019
  - Buy and Hold: 135.2%
  - Strategy: 1,122% (1188 Trades)
    - Huge number of trades might not be realistic (about 1 per week)

## XOP

### XOP Annual Return

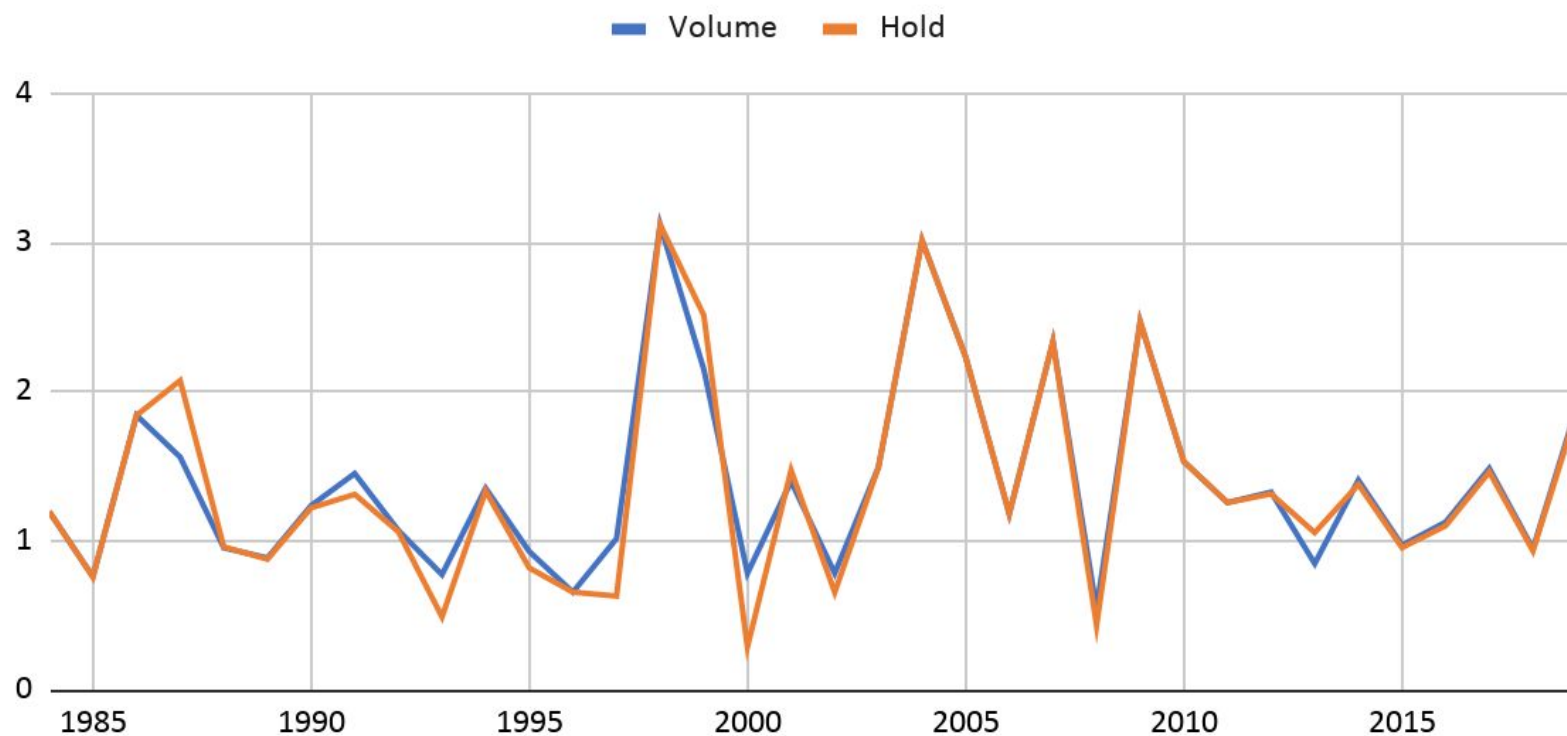


## XOP Cumulative Return



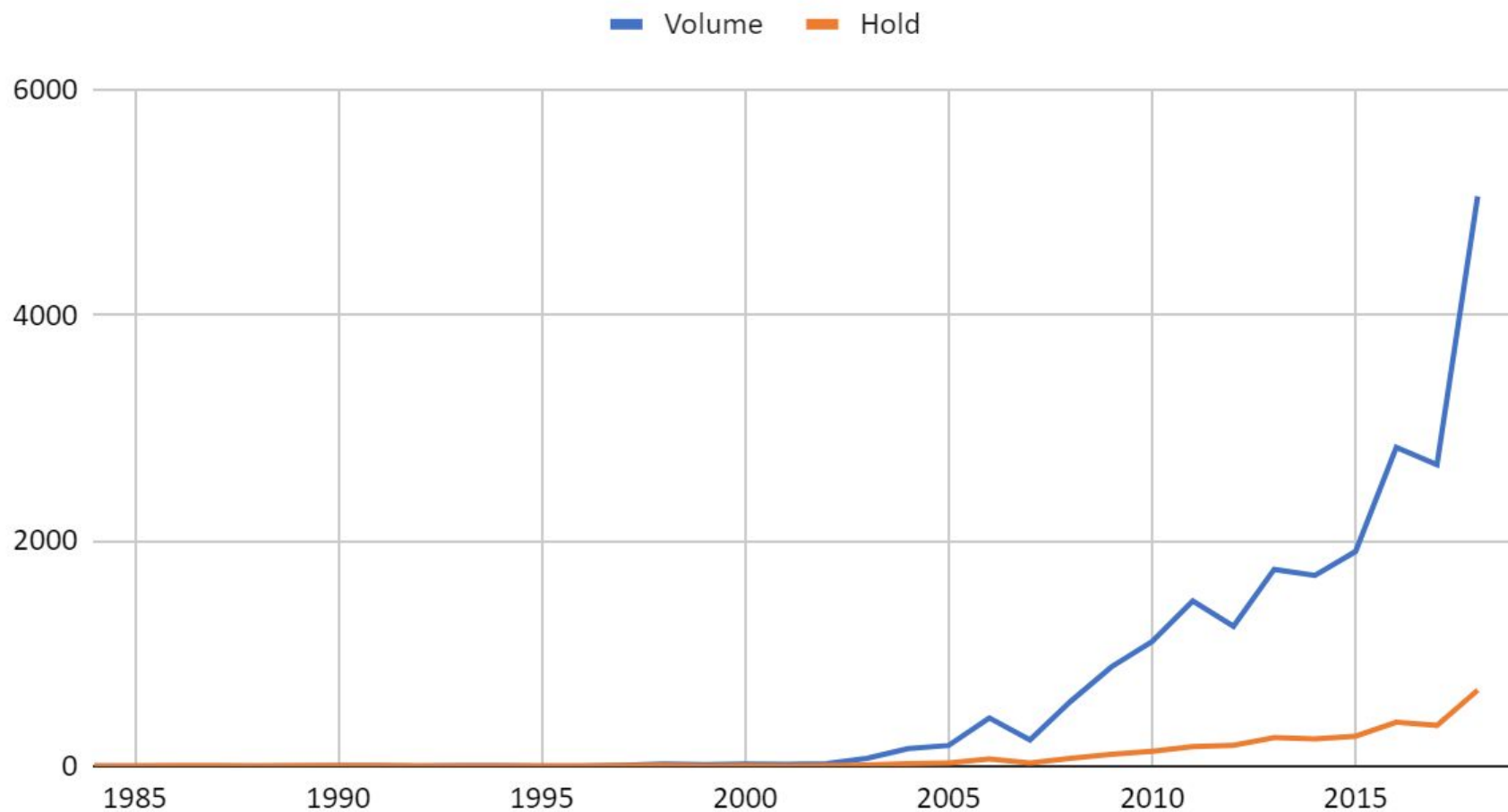
## AAPL

AAPL Annual Return

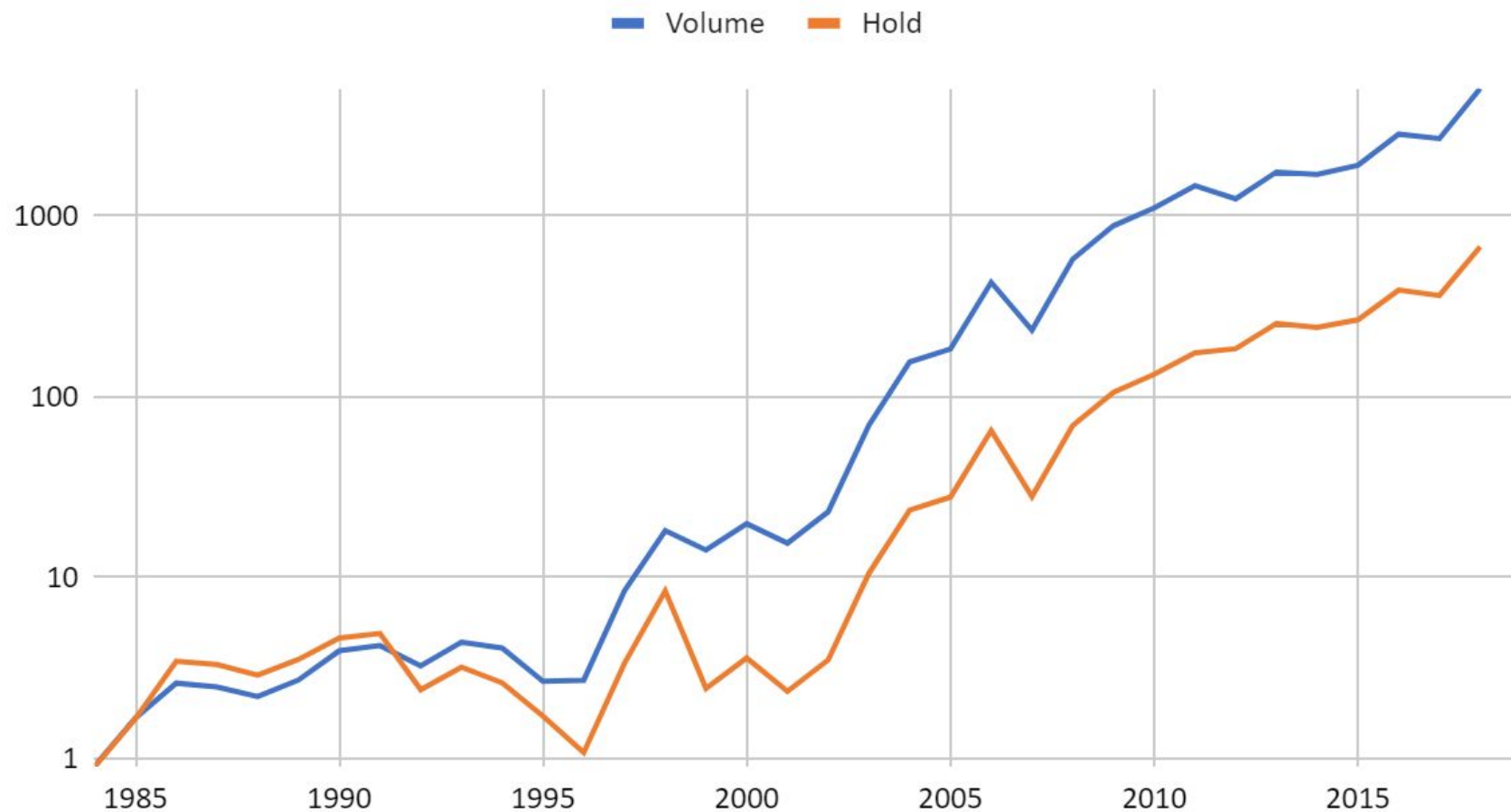




## AAPL Cumulative Return

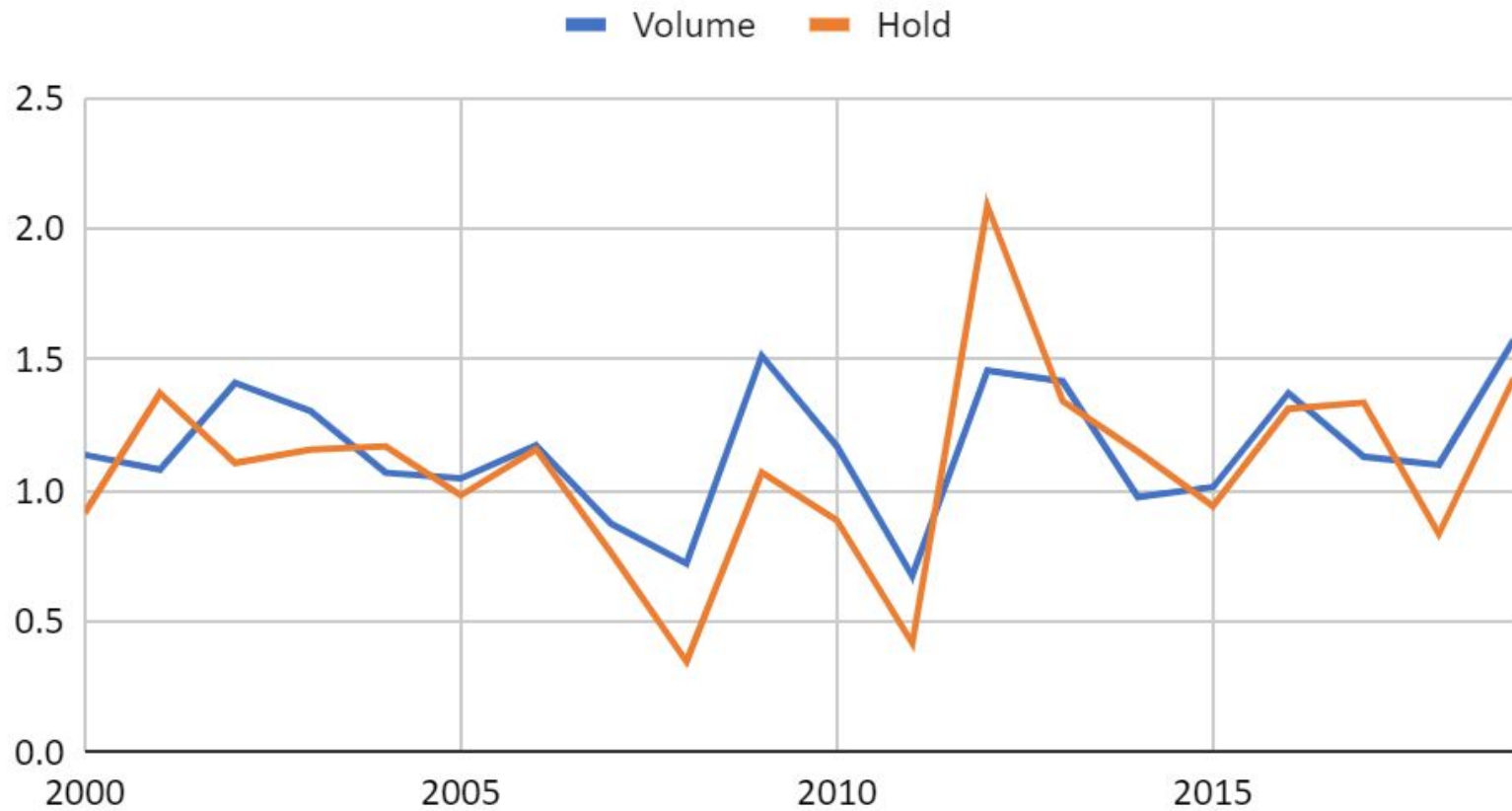


## AAPL Cumulative Return (Log)

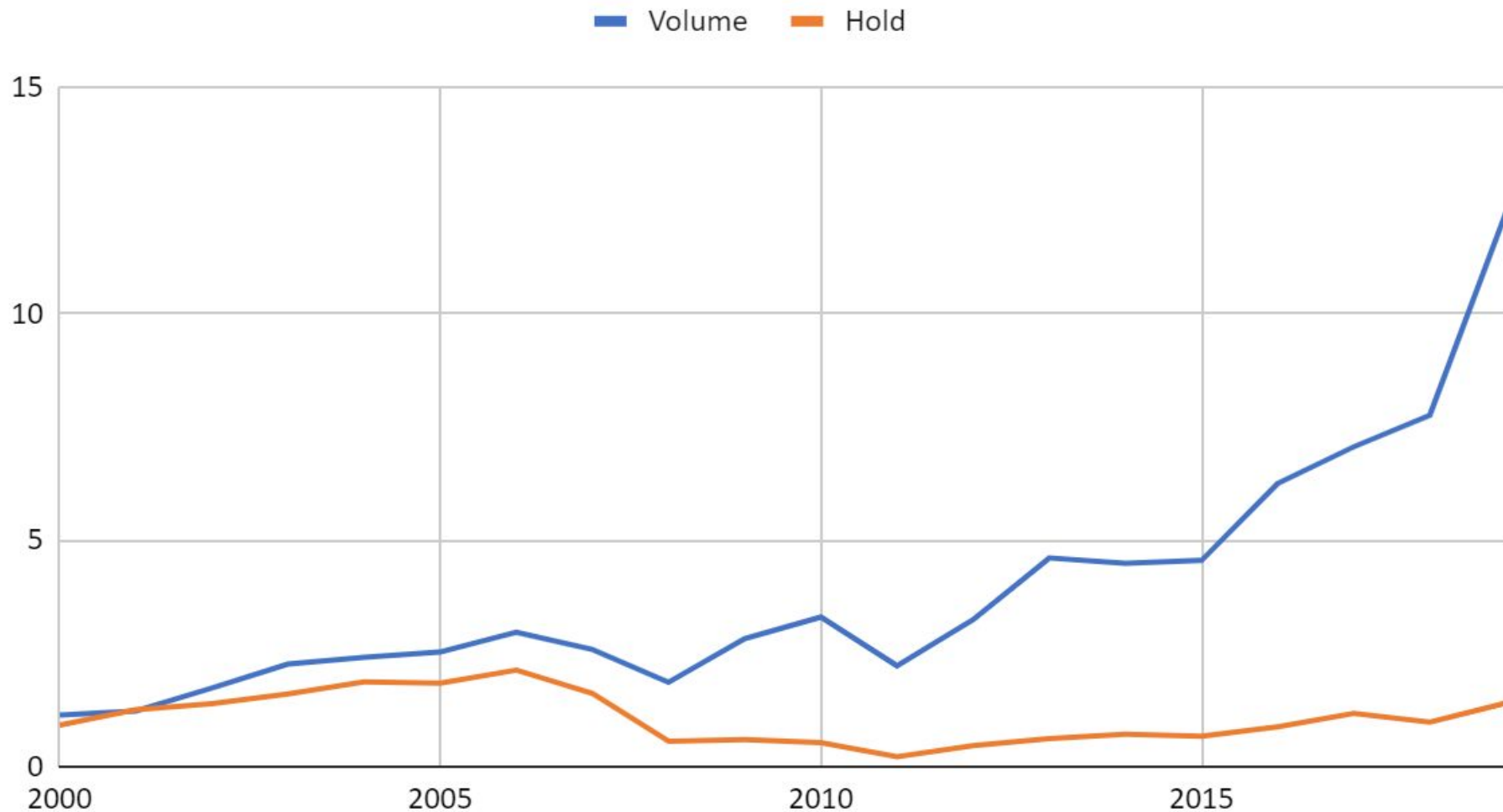


## BAC

### BAC Annual Return

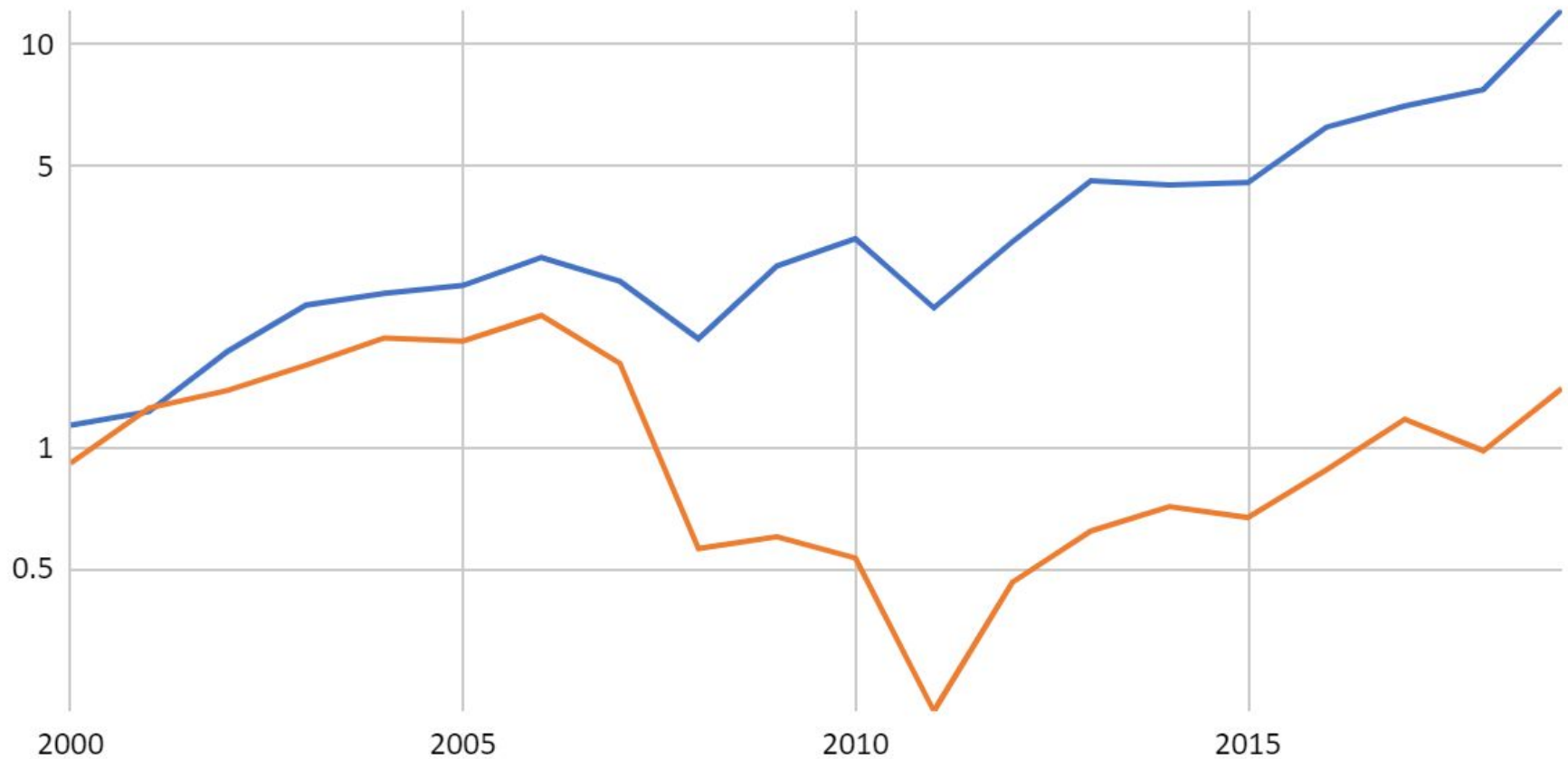


## BAC Cumulative Return



## BAC Cumulative Return (Log)

Volume Hold



**Q&A**

STAT 486 FINAL PROJECT

COVID-19:

# Quarantined Stock Market

...

April 21, 2020

Group 4: Chuyue Huang, Jingxin Mao, Linjing Li, Ye Chen

# Overview

Covid-19 is an ongoing pandemic that has significantly influenced people's lives globally. We will present its impact on U.S. and China stock market so far by analyzing big events and stock indexes.



# Data & Methodology

## Data

We used historical close price of the 8 indexes from 2015/01/01 - 2020/04/15 from Yahoo Finance.

U.S.(5): S&P 500, DJIA, NYSE, NASDAQ, RUSSELL 2000

China(3):  
SSE, CSI 300, CSI 500

## Methodology

We used time series model to fit the data and forecast.

- Train dataset:

2015/01/01 - 2019/04/15

- Test dataset:

2019/04/15 - 2020/04/15

## Analysis

- Plot

compared actual price and predicted price & CI

- Accuracy of ARIMA model fit

ME, RMSE, MAE, MPE, MAPE

# U.S. Stock Indexes

# Trading Curbs happened in history

October 27, 1997

First Trading Curb

Trading curb: the temporary halting of trading

Intentions:

- to rein in excess volatility
- to restore order
- The S&P 500 Index serves as the reference index

March 9,  
2020

Second

March  
12, 2020

Third

March 16,  
2020

Fourth

March 18,  
2020

Fifth

# Potential Causes of Trading Curbs on March 2020

## Failing Oil Prices

On 8 March 2020, Saudi Arabia initiated a price war with Russia.

Triggered by a disagreement over oil-production cuts.

### Implications:

- a 65% quarterly fall in the price of oil
- US oil prices fell by 34%, crude oil fell by 26%, and Brent oil fell by 24.

## Impact of Coronavirus

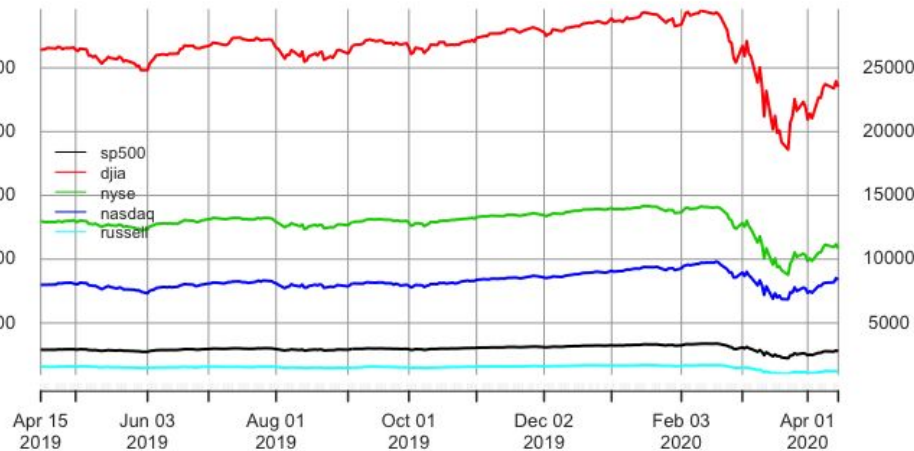
On March 9, 2020, the Dow Jones fell by 7.79%.

On 11 March 2020, the COVID-19 was announced as a pandemic by the WHO.

# Latest 12-Month Price Change

Stock Index Prices

2019-04-15 / 2020-04-15



MAX

S&P 500: 2020-02-19

DJIA: 2020-02-12

NYSE: 2020-01-17

NASDAQ: 2020-02-19

RUSSELL: 2020-01-16

MIN

S&P 500: 2020-03-23

DJIA: 2020-03-23

NYSE: 2020-03-23

NASDAQ: 2020-03-23

RUSSELL: 2020-03-18

Pct drop

S&P 500: -33.925%

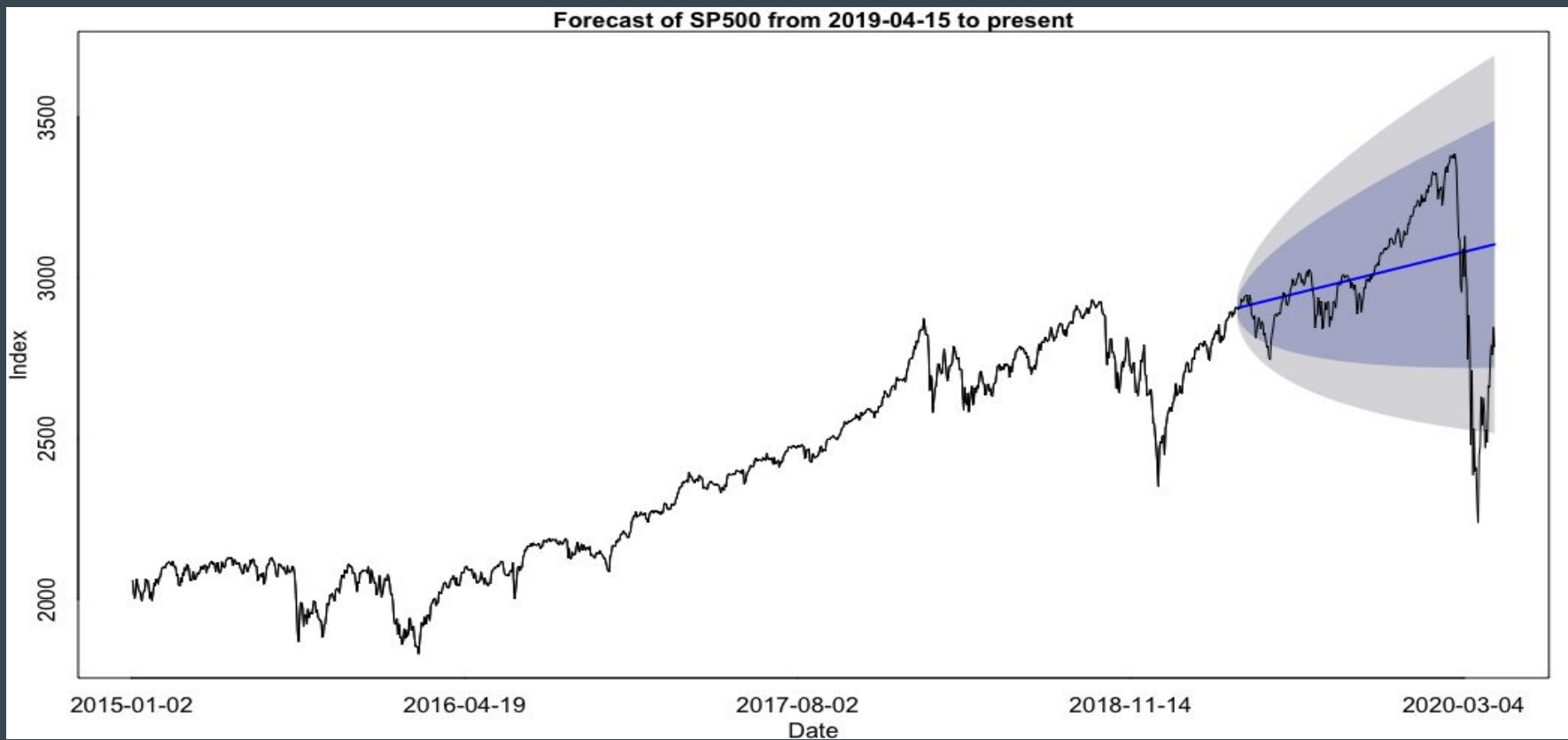
DJIA: -37.086%

NYSE: -38.114%

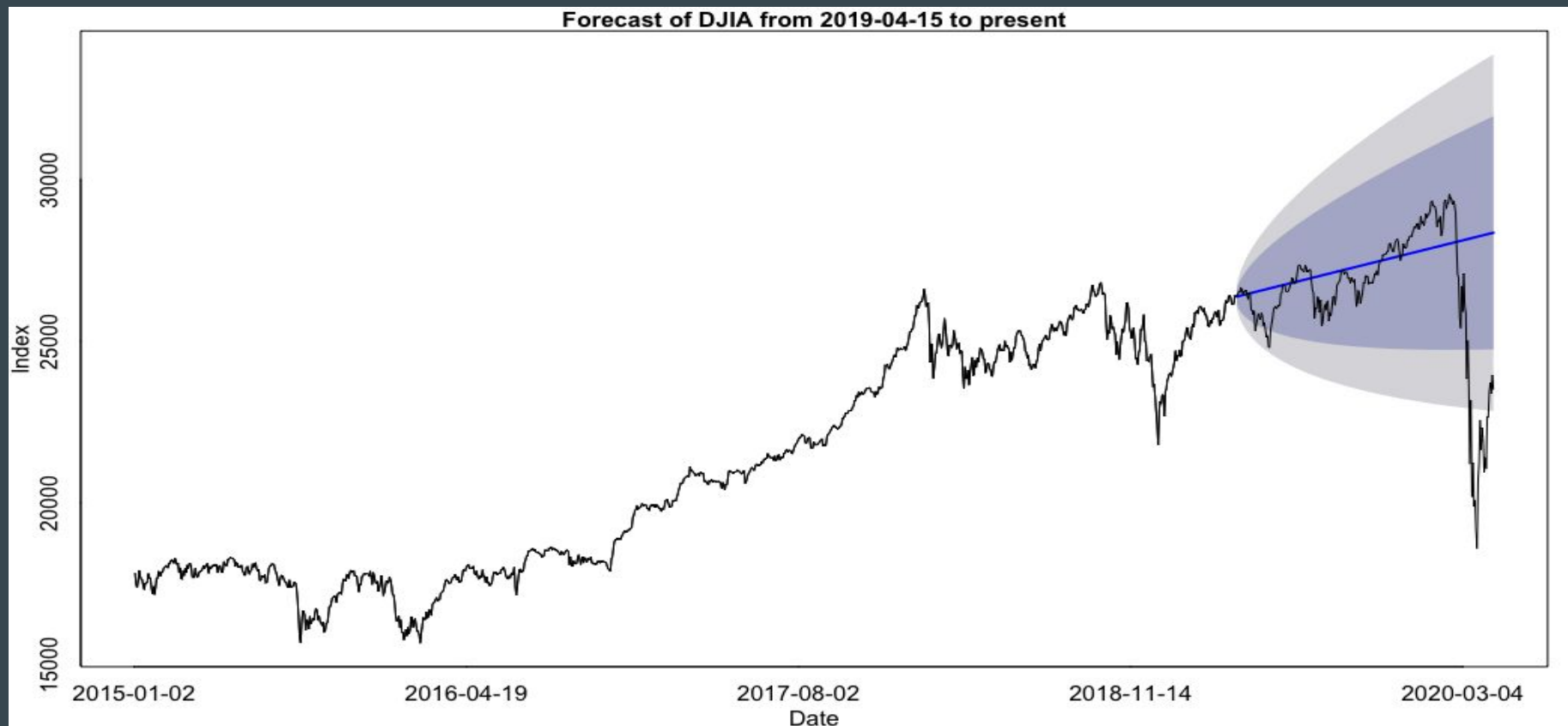
NASDAQ: -30.116%

RUSSELL: -41.875%

# SP500 forecast vs real values

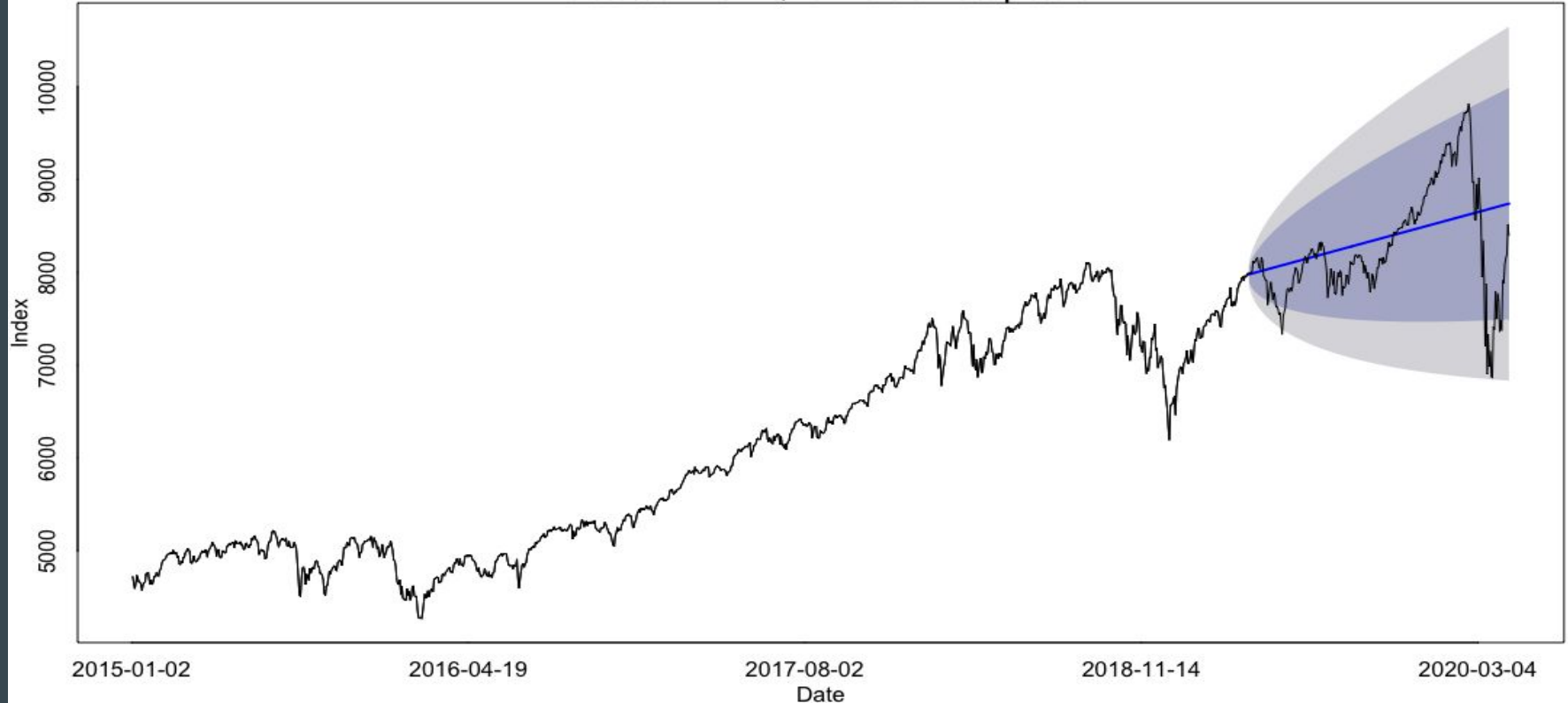


# DJIA forecast vs real values



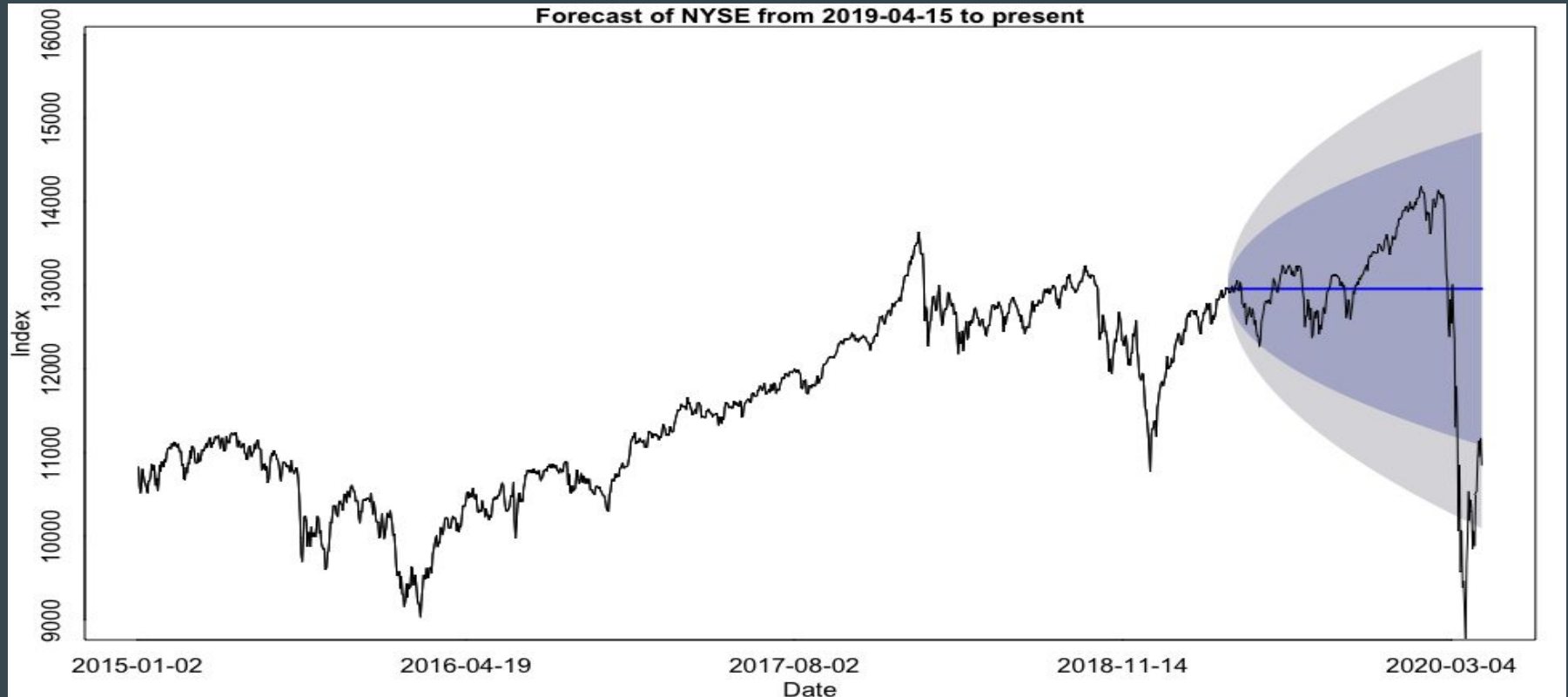
# NASDAQ forecast vs real values

Forecast of NASDAQ from 2019-04-15 to present

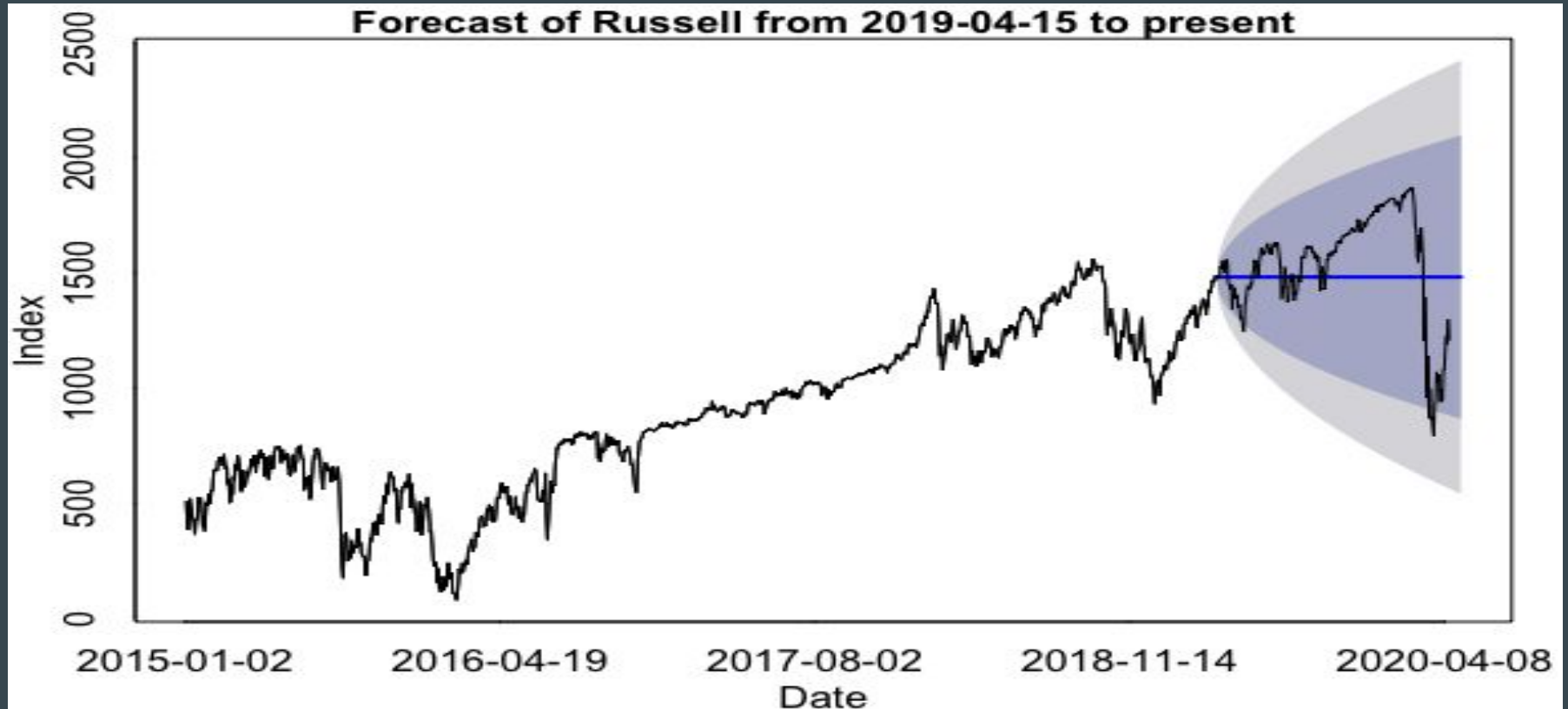




# NYSE forecast vs real values



# RUSSELL forecast vs real values



## SP500

	ME	RMSE	MAE	MPE	MAPE
Training set	-0.00102251	20.05928	13.5929	-0.00585378	0.5836453
Test set	-9970.204	9972.368	9970.204	-336.09	336.09

## DJIA

	ME	RMSE	MAE	MPE	MAPE
Training set	0.01396675	182.7492	124.325	-0.00643767	0.599939
Test set	13620.91	13764.15	13620.91	50.937	50.937

## NASDAQ

	ME	RMSE	MAE	MPE	MAPE
Training set	-0.01225328	62.44613	42.81553	-0.0090513	0.7193307
Test set	-4668.018	4705.598	4668.018	-57.099	57.099

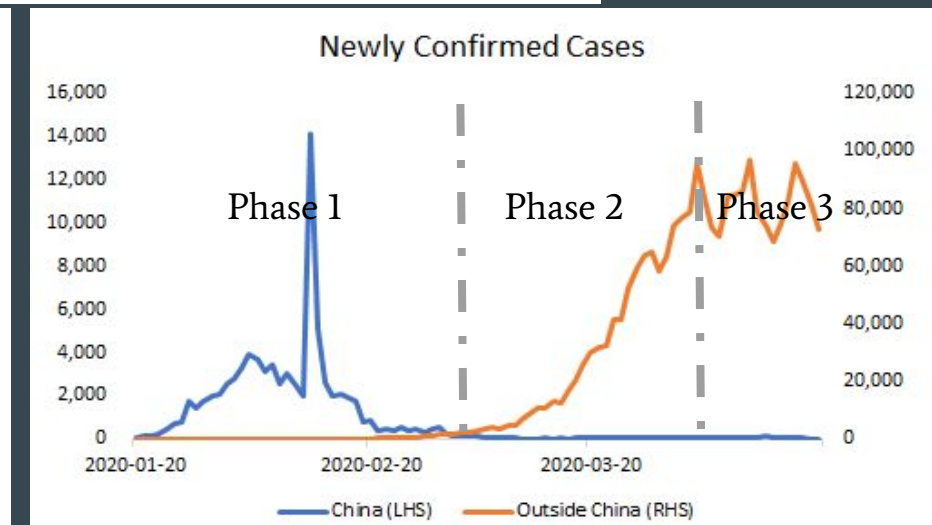
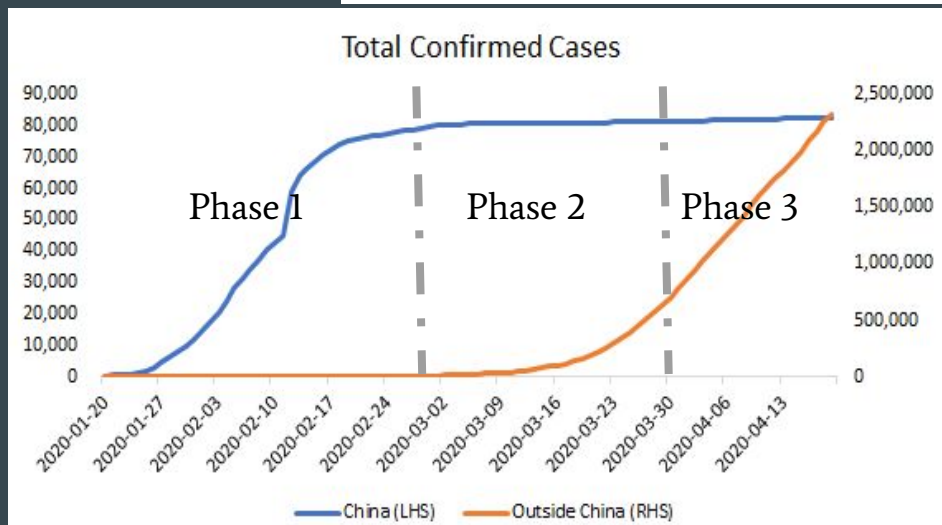
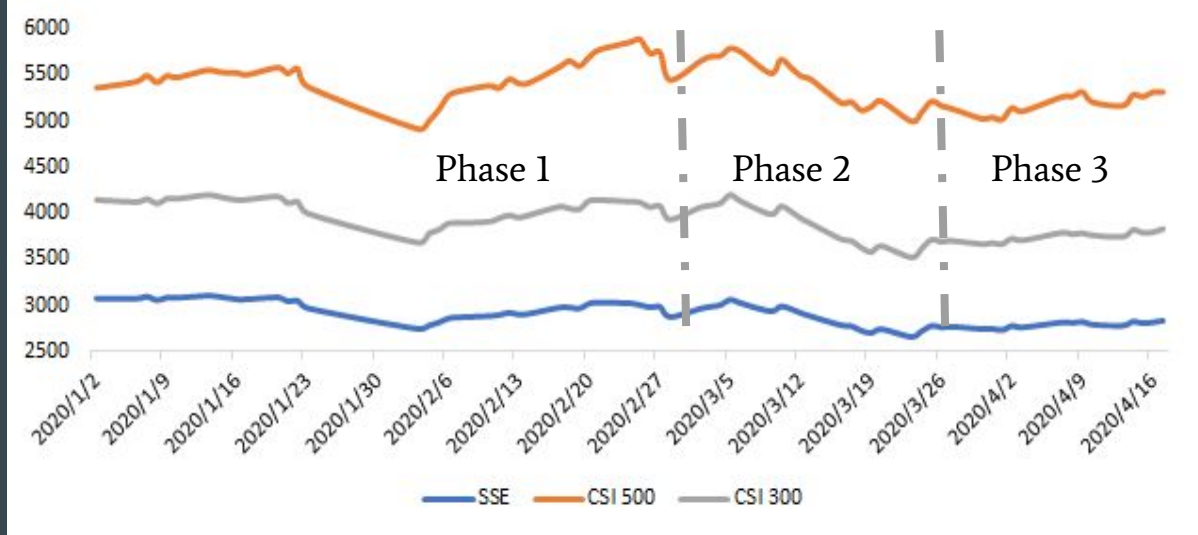
## NYSE

	ME	RMSE	MAE	MPE	MAPE
Training set	1.965182	91.19498	64.55772	0.01323202	0.5750405
Test set	-84.847	1028.366	649.811	-1.423	5.533

## RUSSELL

	ME	RMSE	MAE	MPE	MAPE
Training set	1.034042	34.12674	23.59222	-0.2246203	4.157846
Test set	60.845	235.715	186.088	1.18	13.162

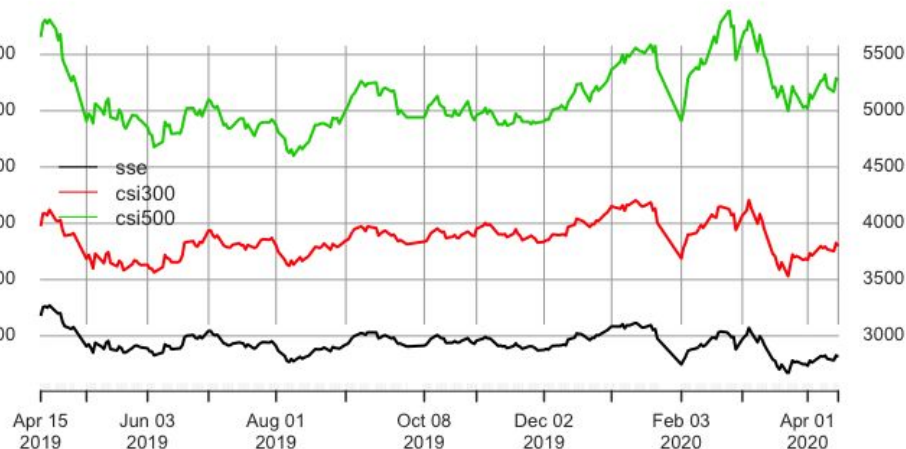
# China Stock Indexes



# Latest 12-Month Price Change

Stock Index Prices

2019-04-15 / 2020-04-15



MAX

SSE: 2020-01-13

CSI 300: 2020-03-05

CSI 500: 2020-02-25

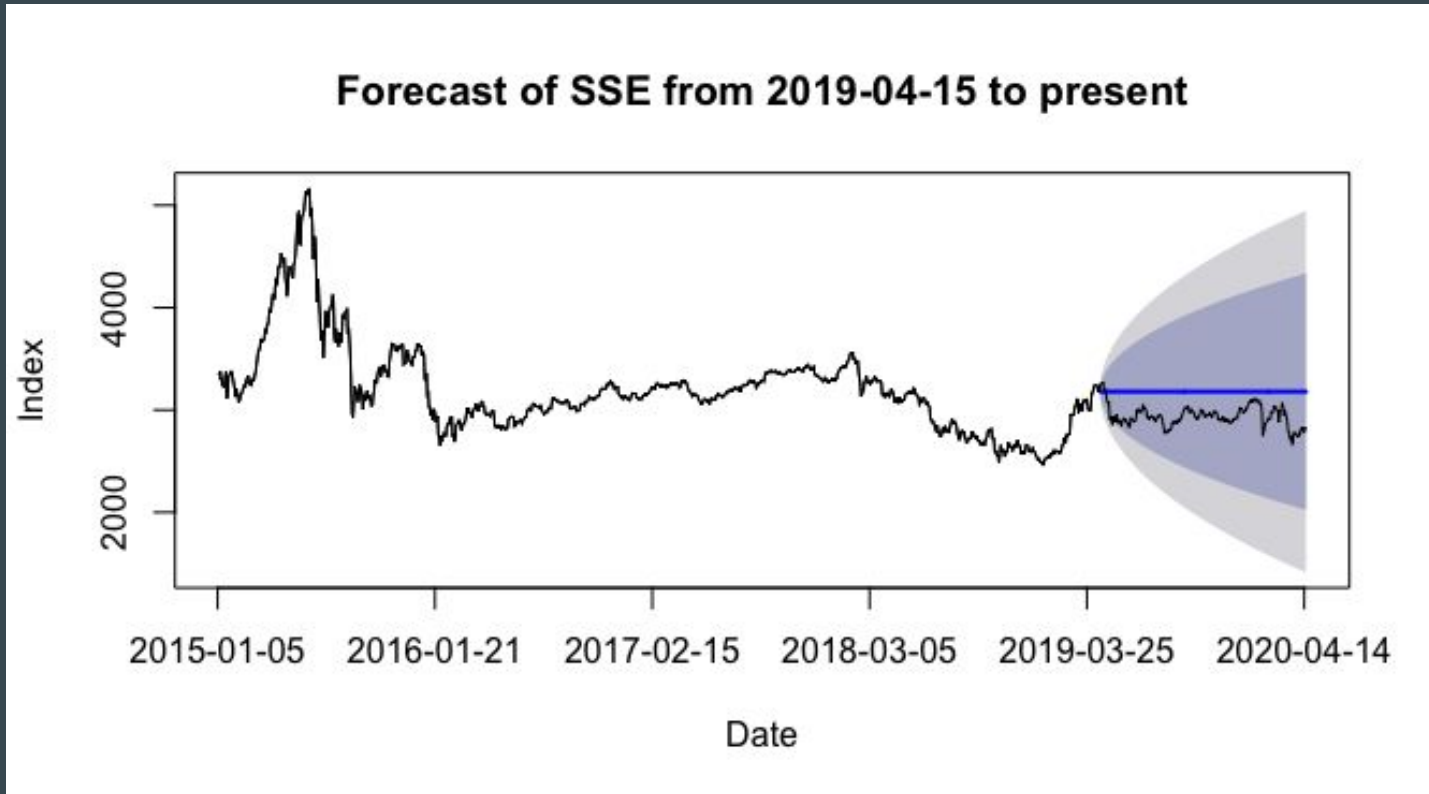
MIN

SSE: 2020-03-23

CSI 300: 2020-03-23

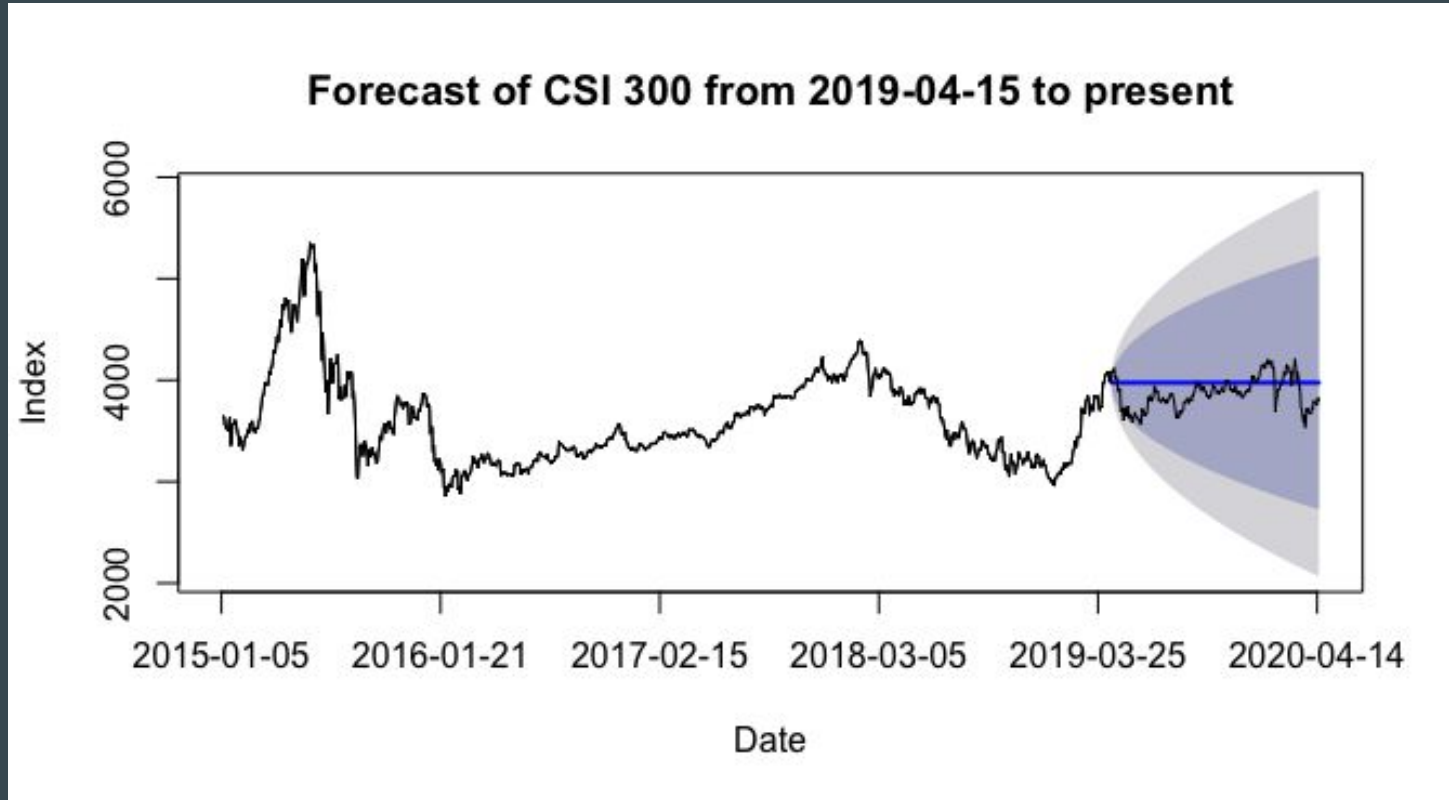
CSI 500: 2020-02-03

# SSE forecast vs real values

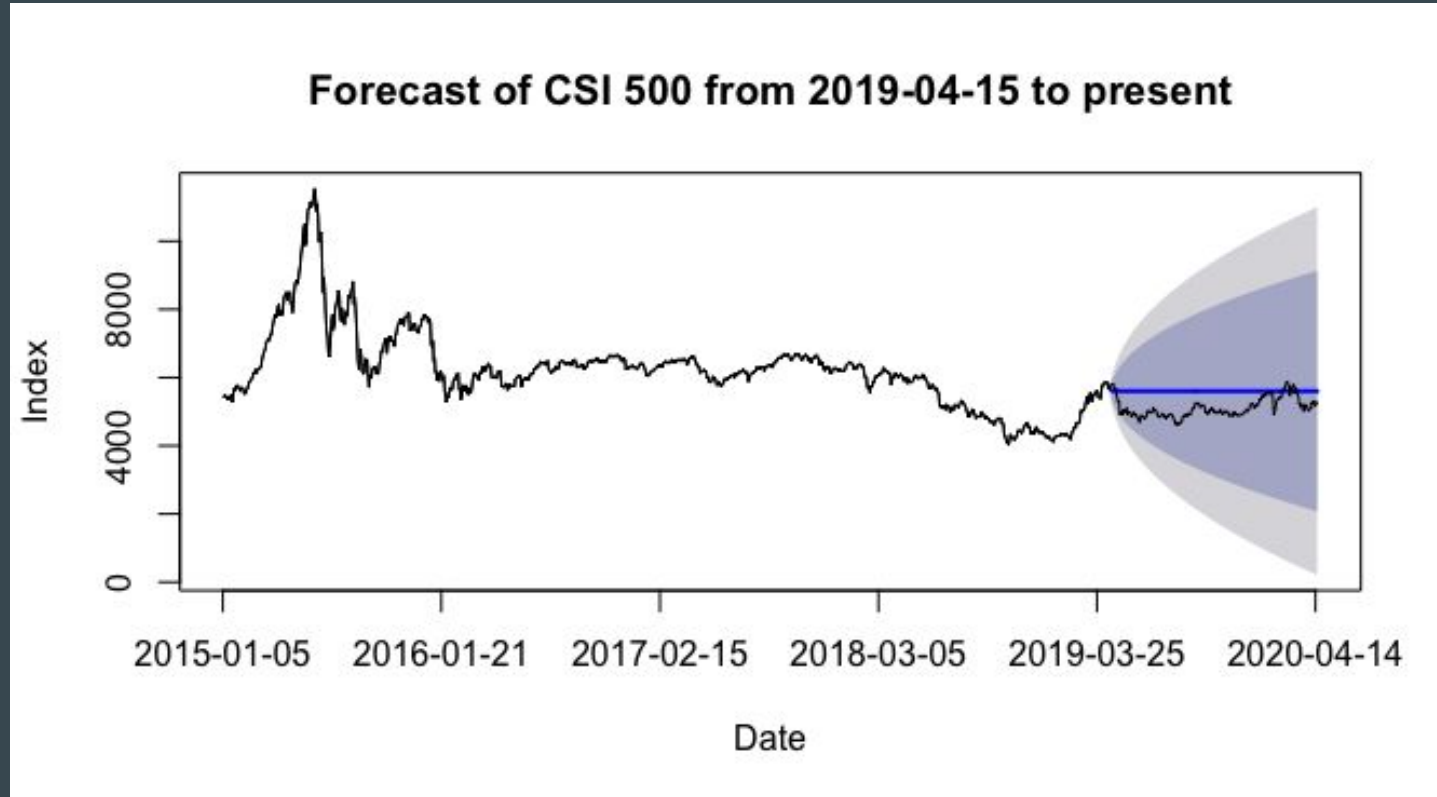




# CSI 300 forecast vs real values



# CSI 500 forecast vs real values



SSE

	ME	RMSE	MAE	MPE	MAPE
Training set	-0.1468153	52.84971	33.26426	-0.0152663	1.015321
Test set	-240.922	261.468	244.128	-8.326	8.425

CSI 300

	ME	RMSE	MAE	MPE	MAPE
Training set	0.3050281	58.58796	38.31997	-0.00357701	1.054531
Test set	-105.748	187.205	155.219	-2.897	4.097

CSI 500

	ME	RMSE	MAE	MPE	MAPE
Training set	0.1350418	130.7629	83.46534	-0.00986904	1.300937
Test set	-484.387	562.014	507.701	-9.793	10.196

# Conclusion

## Global stock market

- Cause: Price war
- Consequence: Trading curbs

## US stock market

- Real stock values drop out of the predicted range

## China's economy

- COVID-19 outbreak generate two waves of economic shocks

**THANK YOU**

**Any Questions?**



# Crude Oil Prices vs Oil Company Performance

Group 5  
Alexander Kalai, Ismail Herrera,  
Alejandro Torrebiarte



# Goals

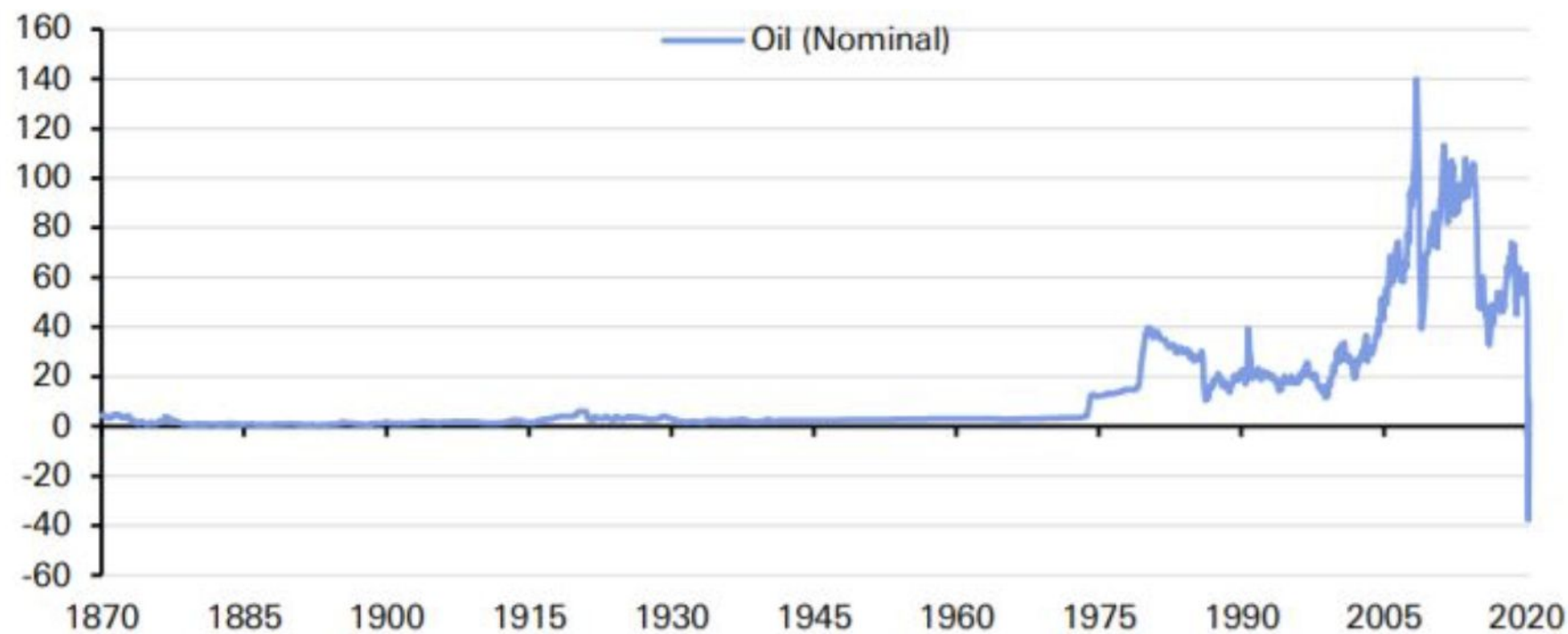
- Find strength of the relationship between crude oil prices and the stock price of large oil companies.
- Determine which financial ratios from oil companies have the most significant relationship with oil prices.
- Run regressions against multiple variables to see if any of them had a stronger relationship than oil price.

# Context

- Oil prices hovered between \$2-3 until the early 1970s
- In March of 1971, the Texas Railroad Commission lost their power as the controller of production to the recently formed OPEC.
- Oil prices rapidly started to rise because of various factors including the Arab oil embargo, leading to an average price from 1970-present of \$34.77/b.
- Recently the coronavirus pandemic caused crude oil futures to momentarily go negative for the first time in history.

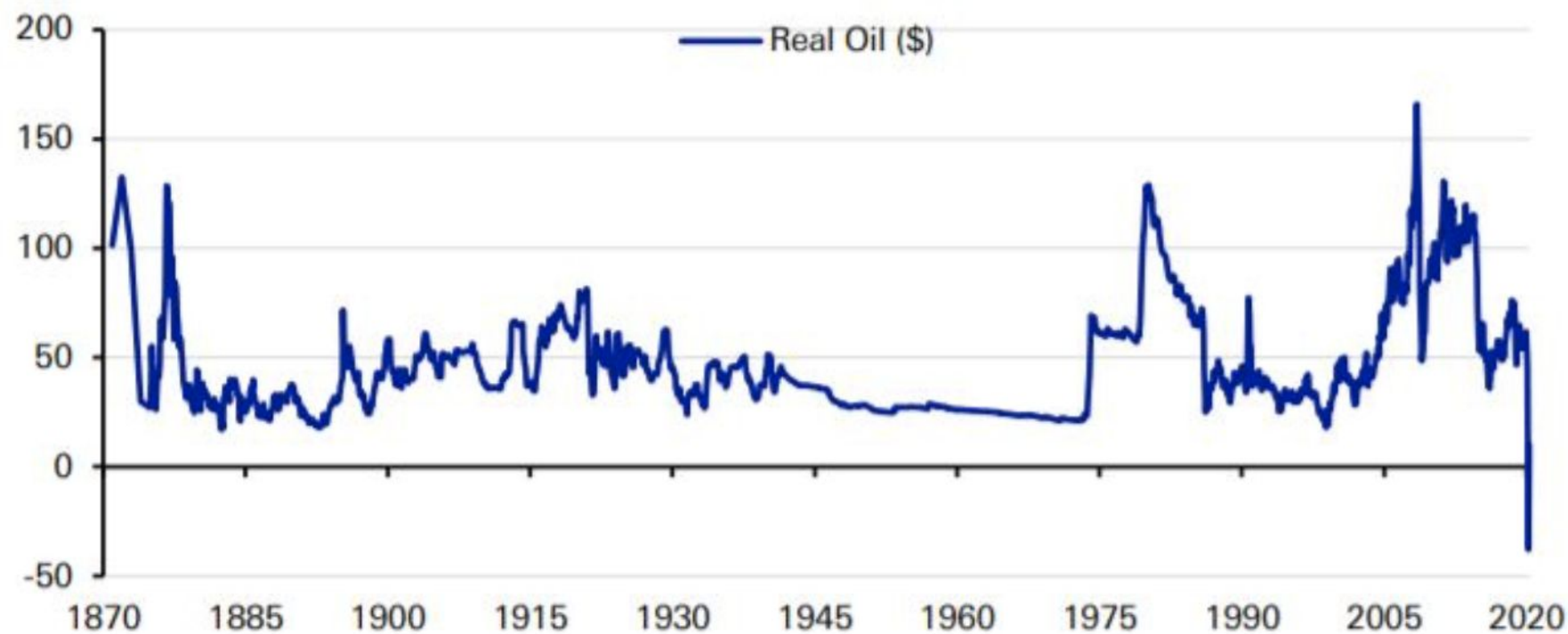


Figure 1: The cost of a barrel of oil in nominal terms



Source : Deutsche Bank, Global Financial Data

Figure 2: The cost of a barrel of oil in real USD terms



Source :Deutsche Bank, Global Financial Data

# Methodology

- Extracted oil price data from the U.S. Energy Information Administration database, using daily spot prices for WTI crude oil.
- Using WRDS, accessed data from CRSP, Compustat and Financial Ratios Suite in order to get data for stock prices and financial ratios.
- Cleaned data to exclude companies with negative stock prices and account for stock splits.

# Results

Plot of Oil and ExxonMobil



Plot of Oil and ConocoPhillips



# First Simple Linear Regression

Stock Price as a Function of Oil Price			
	Estimate	Std. Error	Pr(> t )
Intercept	2.906	0.148	<2e-16
Oil Price	0.859	0.0028	<2e-16

# Stepwise Regressions

- Forward Selection Testing
- Backward Elimination
- Sequential Replacement
- All methods selected oil price as the best predictor
- To clarify, all price ratios are based on the most recent publicly available quarterly SEC filing at the time.
- As a result, there is no risk of look-ahead bias or artificial multicollinearity.



# Stepwise Regression Comparison

	Book to Market	Enterprise Value Multiple	P/E (Diluted, Incl. EI)	Price to Sales	Net Profit Margin	Return on Assets	Return on Equity	Capitalization Ratio	Total Debt to EBITDA	Total Debt to Equity	Price to Book	Price of Oil
1 Variable												***
2 Variables								***				***
3 Variables	**			***				***				***
4 Variables	**			***		***		***				***
5 Variables	**			***		***		***		***		***
6 Variables	***	**		***		***		***		***		***
7 Variables	***	*		***	*	***		***		***	**	***
8 Variables	***	***	*	***	**	***	*	***		**	*	**

**\*= Sequential Replacement, \*\*=Backward Elimination, \*\*\*=Forward Selection Testing**

# Capitalization Ratio

$$\text{Capitalization Ratio} = \frac{(\text{Long Term Debt})}{(\text{Long Term Debt} + \text{Shareholders' Equity})}$$

# ANOVA Testing

- An ANOVA test comparing a regression on price of oil alone versus price and capitalization ratio resulted in a p-value of  $<2.2e-16$ .
- A similar ANOVA test comparing a regression on price and capitalization ratio versus price, capitalization ratio, and Price to Sales ratio resulted in a p-value of  $<2.2e-16$ .
- We continued performing these ANOVA tests for the 6 variables that the stepwise regressions had a consensus on and all had significant p-values.

# Regression By Individual Company

- We achieved extremely high adjusted R-squared values on an individual company basis.
- The adjusted R-squared was 0.7807 for the model including all companies in our set.
- The adjusted R-squared using the same model were 0.9338, 0.9212, 0.9722, 0.9539, and 0.9386 for Chevron, ExxonMobil, ConocoPhillips, Occidental, and Apache, respectively.

# Potential Reasons for Difference

- Diversification of the Company
- Commodity Swaps
- Natural Gas Exposure and other commodities
- Upstream vs Midstream vs Downstream Concentration
- Intercept of individual fit captures these differences

# Beat the Market: Will We Succeed?

**Group 6: Daphne (Jing) Manning, Karen  
Yuan, Joanne Zhou**

April 23, 2020

## **Disclaimer:**

**Past performance does not guarantee future returns.**

**All investing has risks associated.**



## Executive summary

### Goal

To analyze how different investing strategies perform in the long-term by backtesting their performances to the S&P 500 index. We evaluate the strategies based on long-term return (alpha) and volatility (beta).

### Findings

Each investing strategy has its pros and cons

The core-satellite strategy allows for above-average return and below-average risk.

## General Methodology of Backtesting

①

### Determine

the general investing strategy to look at, typically from past projects

②

### Decide

the appropriate time period of backtest and the selection of data to use

③

### Design

the backtest via data manipulation from the available data sources

④

### Derive

the trend from the results and seek to explain the correlation, if any were found

⑤

### Depict

the trends by multiple data visualization



# Seeking Alpha: Market Benchmarks

1970-2019

## Key Statistics

**< 1**

Turnover of portfolio  
per year (FTDOY)

**>2**

Criteria for selection  
under each strategy

**~500**

Pool of stocks our  
model will select  
from (S&P 500  
Index)

**50**

Years of backtest

**4**

Strategies Analyzed

**9.58%**

Equal weighted  
price return of the  
S&P 500 Index

**7.42%**

Price weighted price  
return of the S&P  
500 Index

**35%**

Assumed short-term  
capital gains tax  
(STCG)

**15%**

Assumed long-term  
capital gains tax  
(LTCG)

# Strategies

**01**

---

**Value  
Investing**

**02**

---

**Momentum  
Investing**

**03**

---

**Buy and  
Hold with  
Asset  
Allocation**

**04**

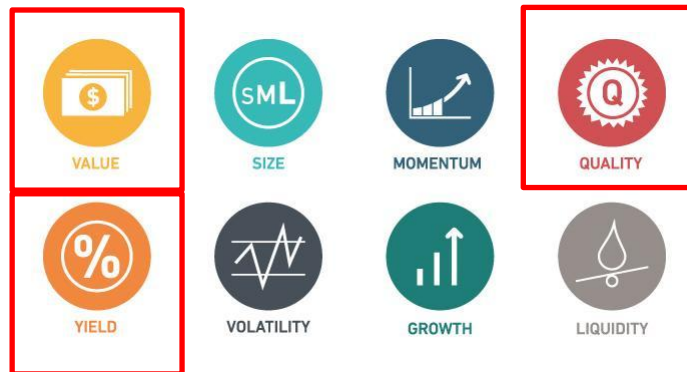
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**Core-Satellite  
Model: Blend  
of Strategies**

# Background of Value Investing

- Investment strategy that involves buying securities that appear to be underestimated by the stock market
- **Warren Buffett**
  - Emphasized need to buy stocks from companies that show strong fundamentals and growth potential
- **Graham and Doddsville**
  - The lower the price of the security relative to its intrinsic value, the higher the “Margin of Safety” would be
  - With time, market value and intrinsic value will converge

## Value Portfolio



Source

Artwork:

<https://www.msci.com/www/blog-posts/understanding-factor-exposures/0858256064>

## Key valuation considerations for selection criteria

A

### Price to Book

$$\frac{\text{Market Capitalization}}{\text{Book Value}}$$

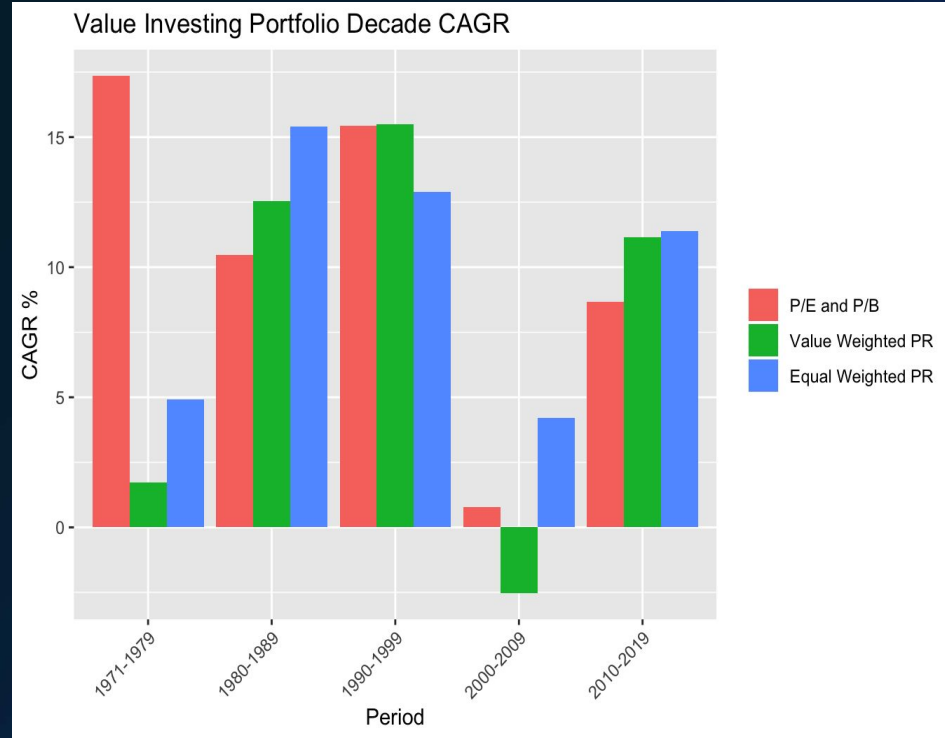
$$\frac{PRCC\_F \times CSHO}{AT-LT}$$

B

### Price to Earnings

$$\frac{\text{Market Price Per Share}}{\text{Earnings Per Share}}$$

$$\frac{PRCC\_F \times CSHO}{NI}$$





## Value Investing

*“Be fearful when others are greedy and to be greedy only when others are fearful.”*

*-Warren Buffett*



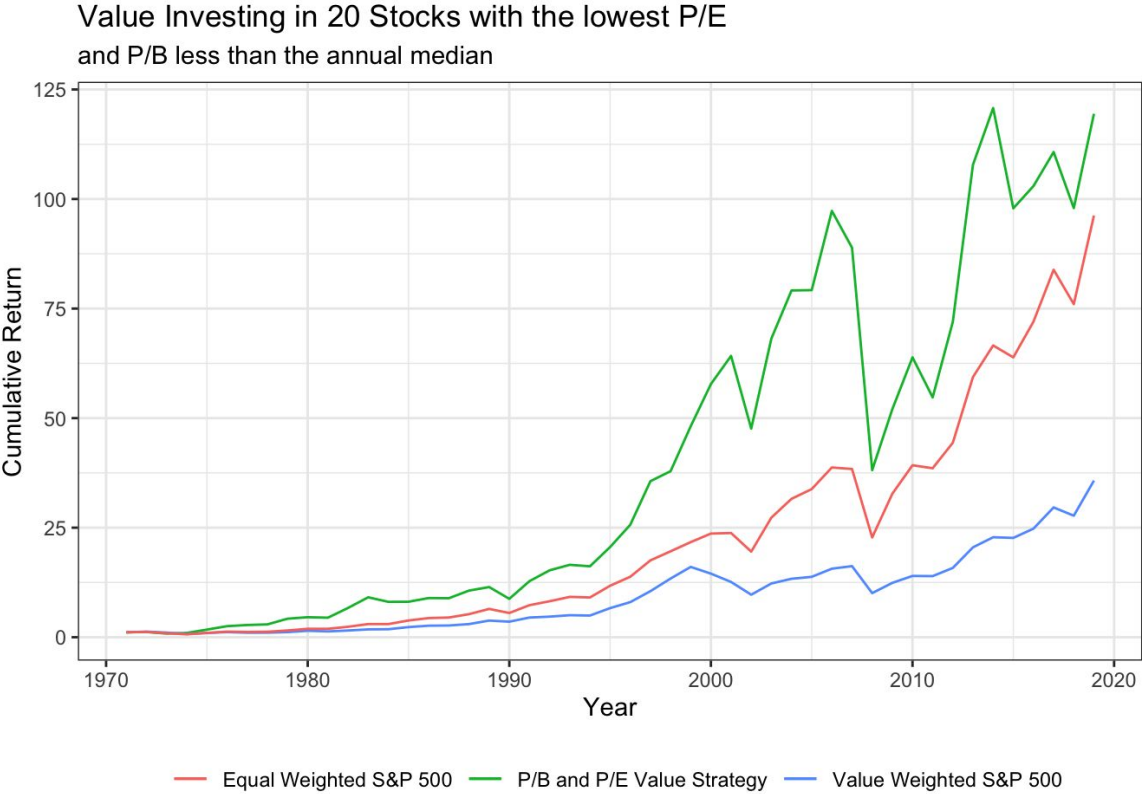
## Methodology

1. Limit the S&P 500 stocks to those with a P/B ratio less than the annual median
2. Find the 20 stocks with the lowest P/E ratio, arguably the stocks with the most value
3. Invest in the 20 stocks for next year by purchasing on FTDOY and hold until the LTDOY
4. Calculate the annual return for the portfolio, assuming the stocks will be equal-weighted

## Economic Impacts

Period	P/E and P/B Criteria	Value Weighted PR	Equal Weighted PR
1971-1979	17.37	1.73	4.93
1980-1989	10.48	12.54	15.40
1990-1999	15.44	15.49	12.88
2000-2009	0.78	-2.54	4.20
2010-2019	8.66	11.16	11.38

Cumulative Returns and N-Year CAGR



	N-Year CAGR *N=49	Volatility
Value Investing	10.2%	+/- 24.3%
Value Weighted S&P 500 PR	7.57%	+/- 16.7%
Equal Weighted S&P 500 PR	9.77%	+/- 18.1%

# Strategies

01

---

**Value  
Investing**

02

---

**Momentum  
Investing**

03

---

**Buy and Hold  
with Asset  
Allocation**

04

---

**Core-Satellite  
Model: Blend  
of Strategies**

# Selecting Criteria for S&P 500 High Momentum Low Volatility

## 1

### Low Volatility

1a. **Find** volatility of all S&P 500 constituents base returns

1b. **Eliminate** stocks that are more volatile than the median

## 2

### High Momentum

2a. **Use the Max-Median** rule to find median of daily returns for each stock from the low volatile stocks

2b. **Rank** from largest median to least

2c. **Select** the top 20 stocks from the ranked list, as 20 was the "optimal" portfolio size found in a previous study

Methodology

### High Momentum Low Volatility



VALUE



SIZE



MOMENTUM



QUALITY



YIELD



VOLATILITY



GROWTH

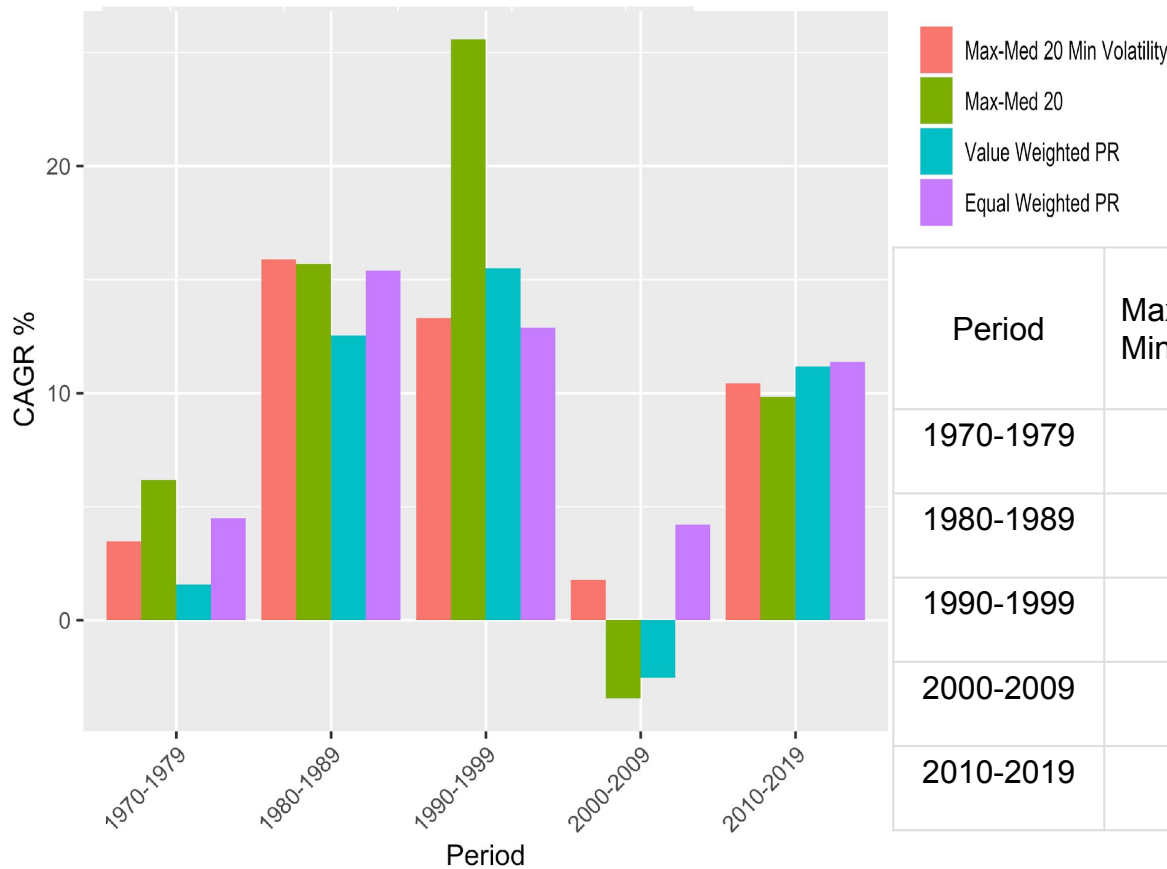


LIQUIDITY



Max-Median Decade Trend Changes 1970-2019

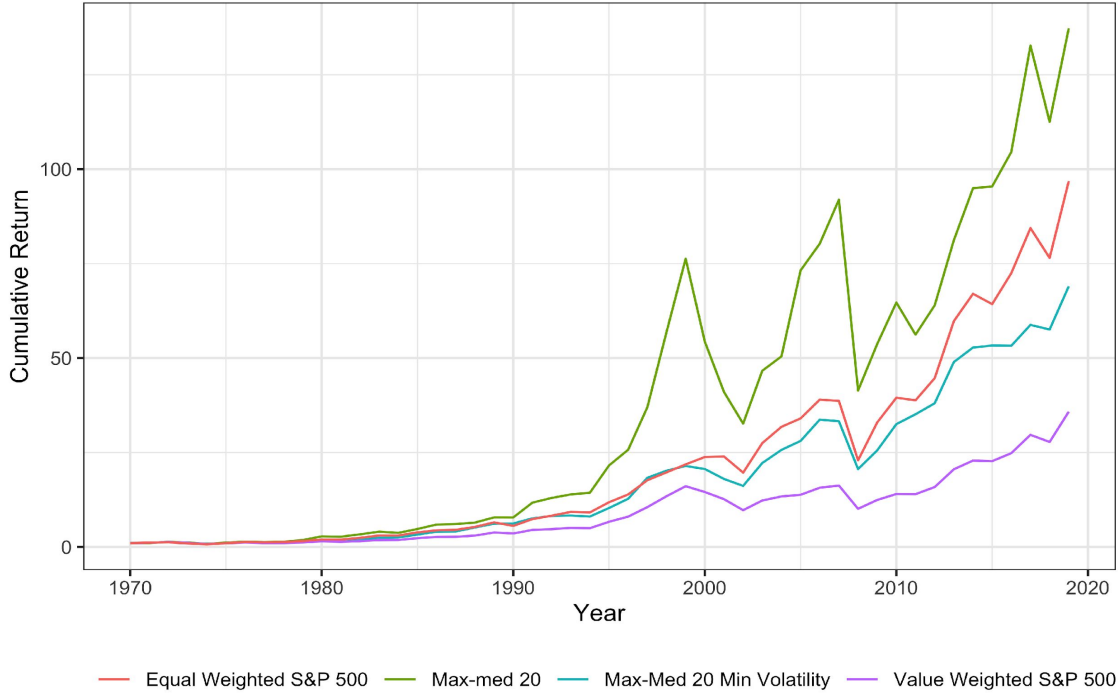
Max-Median and Max-Median+ Min Volatility Portfolios Decade CAGR



Period	Max-Med 20 Min Volatility	Max-Med 20	Value Weighted PR	Equal Weighted PR
1970-1979	3.47	6.18	1.57	4.50
1980-1989	15.90	15.69	12.54	15.40
1990-1999	13.30	25.58	15.49	12.88
2000-2009	1.78	-3.45	-2.54	4.20
2010-2019	10.44	9.84	11.16	11.38

Cumulative Returns

Max-Med with Minimum Volatility



	N-Year CAGR *N=50	Volatility
Max-Med 20 Minimum Volatility	8.84%	+/- 16.7%
Max-Med 20	10.35%	+/- 23.8%
Value Weighted S&P 500 PR	7.42%	+/- 16.5%
Equal Weighted S&P 500 PR	9.58%	+/- 17.9%

# Strategies

01

---

Value  
Investing

02

---

Momentum  
Investing

03

---

Buy and  
Hold with  
Asset  
Allocation

04

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Core-Satellite  
Model: Blend  
of Strategies

# Asset Allocation Model

- We assumed rebalancing every year.
- Different allocation is a personal preference based on goals, time, and risk tolerance.



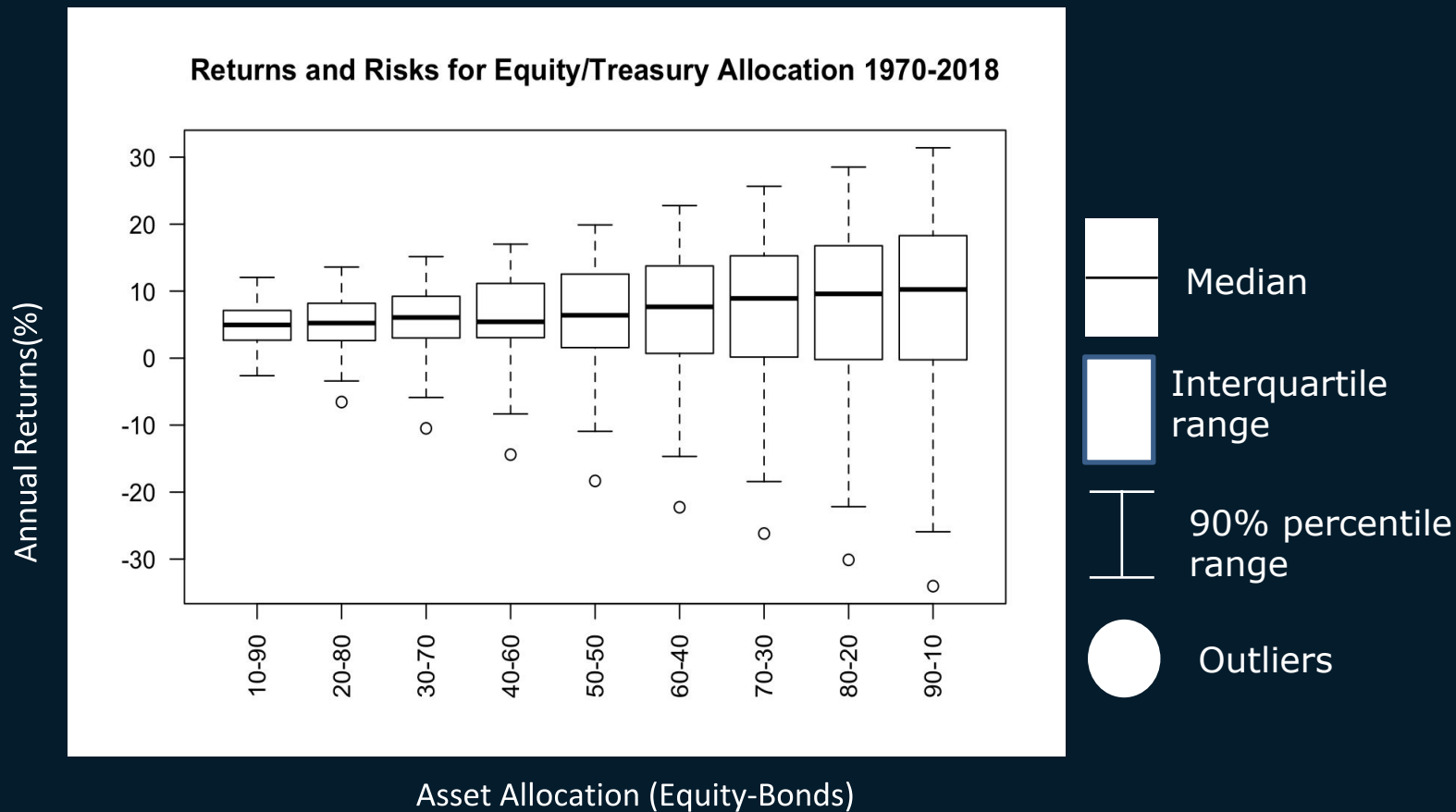
## BONDS

- To simulate the return of bonds: we used the annual Fama French risk free return rate  $R_f$  that was calculated in a previous study involving CAPM (Capital Asset Pricing Model)
- We realize that real-world returns may vary due to the types of bonds chosen (government treasury, municipal, corporate, etc).
- The key point is that bonds provide stabilization and guaranteed returns in a healthy long-term portfolio , allowing for rebalancing.

## EQUITIES

- We chose to use the S&P 500 value weighted price return index as the benchmark of annual equity return.
- While this undermines the power of dividend reinvestment, the model aligns with strategies mentioned earlier.

With different equity/bonds allocations, there is a trend of increased variance and increased median returns when equity allocation increases.

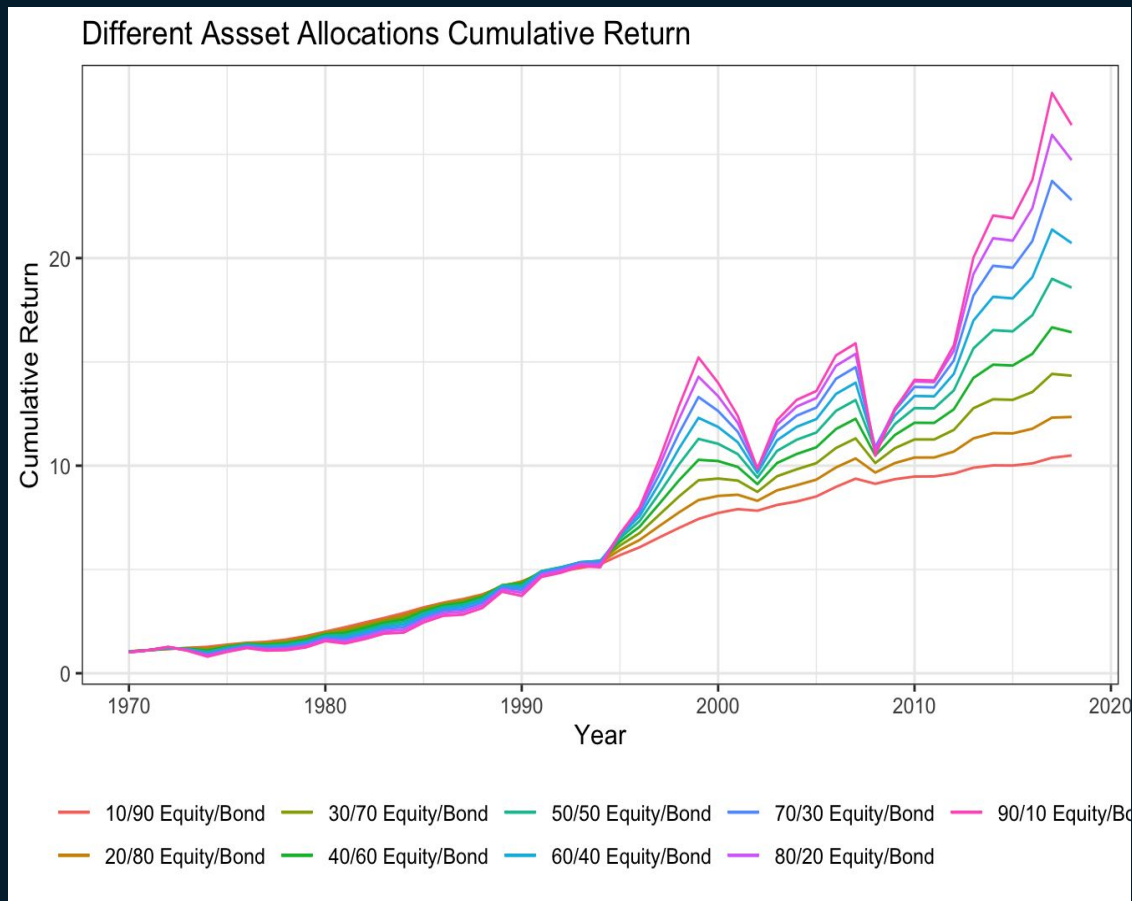


# Cumulative Returns

## Key Trends



- Higher equity allocations outperformed in recent years because of lower interest rates diminished the return of bonds
- Right now: there is a changing environment for treasury bonds as the world moves into largely negative rates environment.
- Future role of bonds is uncertain.

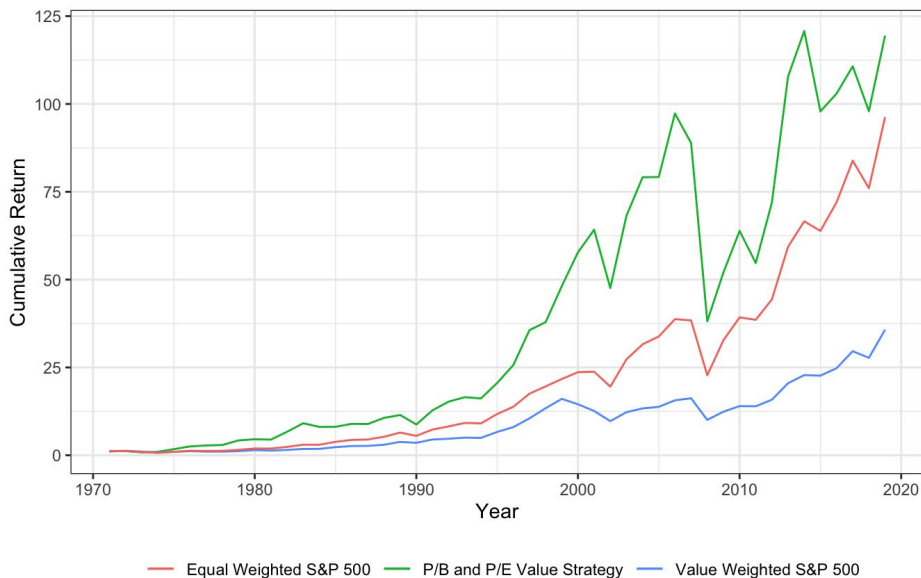


## Benefits of Buy and Hold

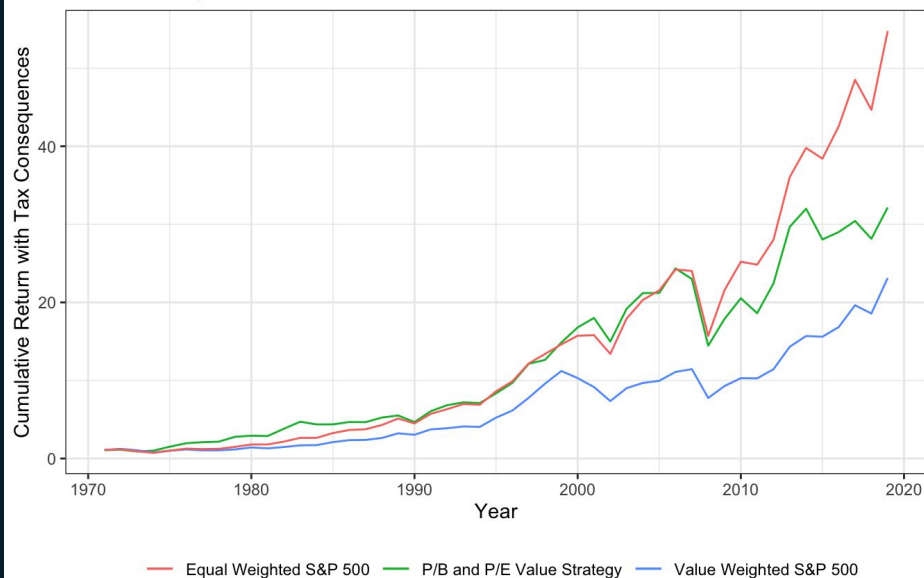
- Easy to implement: very simple and can be done passively (not much guesswork needed)
- Great for taxes: long-term capital dividends and gains have lower taxes
- Do not need market analysis: do not need to worry about market timing and volatility.
- Simple investment vehicles: best buy and hold investment vehicles are low-cost index and ETF funds.

## EFFECTS OF TAX CAUSING DRAG ON RETURNS!

Value Investing in 20 Stocks with the lowest P/E and P/B less than the annual median



Value Investing in 20 Stocks with the lowest P/E and P/B less than the annual median



# Strategies

01

---

Value  
Investing

02

---

Momentum  
Investing

03

---

Buy and  
Hold with  
Asset  
Allocation

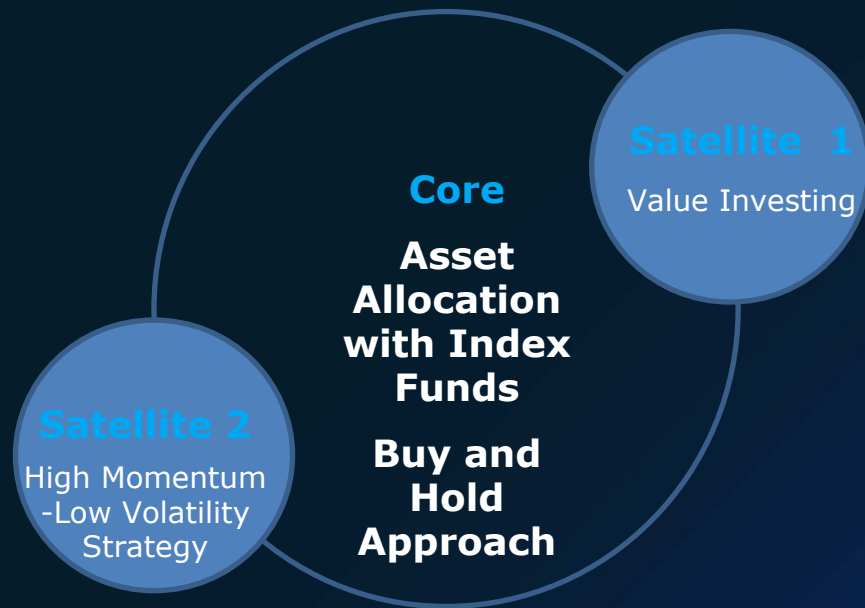
04

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Core-Satellite  
Model: Blend  
of Strategies



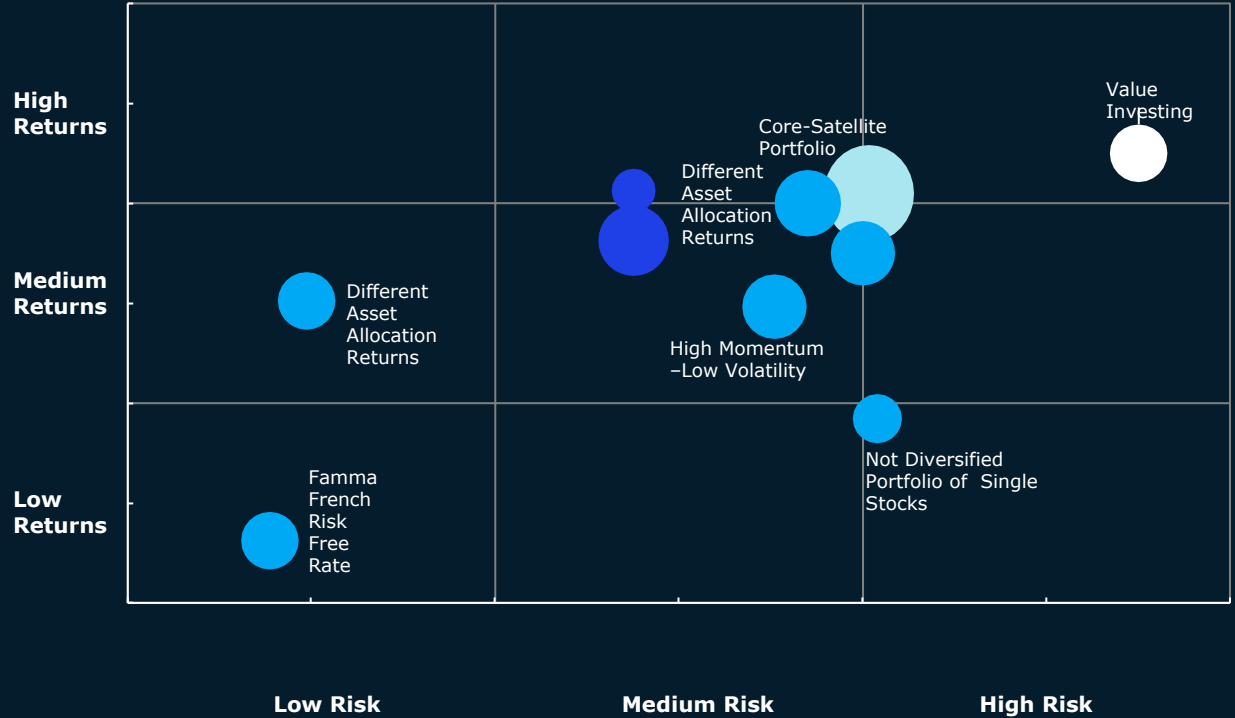
## Core-Satellite Model



## Seeking Continuous Alpha

"Beating the index is challenging year after year"

# Risk/Return of Different Strategies



## Conclusion

**01**

Each investing strategy has its pros and cons.

**03**

Investing mostly with the passive buy and hold approach would allow for a stable core.

**05**

Having different satellites ensures investors are well-diversified.

**02**

Not diversifying investments will result in high risk and low return.

**04**

Satellites that use higher risk investing approaches would allow for the possibility of outperforming the standard market benchmark.

**06**

The core-satellite investing strategy overall allows for above-average returns with below-average risk.

## References

<https://www.investopedia.com/articles/active-trading/070715/pros-cons-passive-buy-and-hold-strategy.asp>

<https://www.thebalance.com/what-is-core-and-satellite-2466544>

<https://www8.gsb.columbia.edu/valueinvesting/about/history>

Previous projects from this semester

## Group 7: STAT 686 Final Project Prospectus

### Stock Indices Pricing under Incomplete Market

Security and derivatives pricing under risk-neutral measure is a well-studied area in financial engineering. Within this theoretical framework, to price a security the risk-free rate and volatility are both regarded as constant, whereas these assumptions are essentially in conflict with the market we are facing every day. The realistic market, however, has more unpredictable factors, making the security pricing extremely difficult --- the rate and volatility also show stochastic patterns.

Therefore, we will seek how stochastic interest rate models and stochastic volatility models which were developed in the 1980s, perform in the Financial crisis period compared with high frequency methods developed in the 2000s.

DATA resource: 2005/01/01-2010/12/31 WRDS (Wharton Research Data Research)  
DOW JONES INDEX and S&P500 INDEX

Stochastic Model:

Interest Rate model: Cox, Ingersoll, Ross model.

[https://en.wikipedia.org/wiki/Cox%E2%80%93Ingersoll%E2%80%93Ross\\_model#Simulation](https://en.wikipedia.org/wiki/Cox%E2%80%93Ingersoll%E2%80%93Ross_model#Simulation)

Volatility model: Heston

[https://pdfs.semanticscholar.org/d5b9/b0ac52fd359eb2ce892b94ea7e8b20cb3b8d.pdf?\\_ga=2.135180913.1710287941.1586837004-235341238.1586837004](https://pdfs.semanticscholar.org/d5b9/b0ac52fd359eb2ce892b94ea7e8b20cb3b8d.pdf?_ga=2.135180913.1710287941.1586837004-235341238.1586837004)

High Frequency Approach:

Risk-free interest rate: Binsbergen(2019)

[https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=3242836](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3242836)

Volatility: Yang & Zhang(2000):

[https://www.jstor.org/stable/10.1086/209650?seq=1#metadata\\_info\\_tab\\_contents](https://www.jstor.org/stable/10.1086/209650?seq=1#metadata_info_tab_contents)

For the high Frequency approaches, we will simulate stock price following geometric Brownian motion, then plug simulated interest rate and volatility into the following equation.

After solving PDE(SDE), we have stock price in discrete time as

$S_t = S_{t-1} \cdot \exp(r - 0.5\sigma^2 + \sigma \cdot \eta_t)$ , where  $\eta_t \sim i.i.d N(0,1)$





# Stock Indices Pricing under Incomplete Markets

Group 7: Xu Chen Seth Kimmel Leila Lei  
Hao Qiu Yutong Su Bojie Zhang

# Background

- Incomplete market
  - Complete market implies unique neutral risk measure => complete/perfect asset price understanding.
  - If interest rate and volatility bring a second noise term and perfect market information is unknown, the market is incomplete.
- Security pricing assumes constant risk-free rate and volatility
  - Stochastic in reality
  - Different methods to model them
- Goal:
  - Combine **stochastic interest rate & stochastic volatility methods** (developed in 1980s) to simulate stock price
  - Compare the results with those produced by **high frequency methods** (developed in 2000s).



# Literature Review/Methods

	Interest model	Volatility model
<b>Stochastic model</b>	<i>Ho Lee model (1986):</i> Stochastic differential equation describes the evolution of interest rates	<i>Jacquier (1994):</i> Stochastic Volatility
<b>High frequency method</b>	<i>Binsbergen(2019):</i> Present risk-free interest estimator without convenience yield.	<i>Yang and Zhang (2000):</i> Present a new volatility estimator with dramatic improvements.

# Data

- Options data from the Chicago Board Options Exchange (CBOE) on two underlying assets to simulate interest rate.
  - S&P 500 Index (SPX)
  - Dow Jones Index (DJX)
- S&P 500 Index to simulate volatility and stock price return

## Training/Testing:

- Use 2010 - 2014 data to train our models
- Simulate 2015-2019 stock price data
- Compare to actual stock prices from 2015-2019

# Methodology

Assume stock price follows Geometric Brownian Motion:

$$\frac{dS_t}{S_t} = rdt + \sigma dW_t \quad (1)$$

Using Ito's Lemma solving SDE, we stock model in discrete time as

$$S_t = S_{t-1} \exp(r - 0.5\sigma^2 + \sigma\epsilon_t) \quad (2)$$

Which implies log return follows Normal Distribution

$$\log\left(\frac{S_t}{S_{t-1}}\right) \sim N\left(\mu - \frac{1}{2}\sigma^2, \sigma^2\right) \quad (3)$$

# Stochastic Volatility Model

R package “stochvol” uses MCMC to simulate stochastic volatility model.

“Centered” version (e.g. Jacquier, Polson, and Rossi, 1994):

$$\begin{aligned} y_t &= e^{h_t/2} \varepsilon_t, & \varepsilon_t &\sim N(0, 1), \\ h_t &= \mu + \phi(h_{t-1} - \mu) + \sigma \eta_t, & \eta_t &\sim N(0, 1), \end{aligned}$$

with  $\mathbf{h} = (h_0, \dots, h_T)$ ,  $\mu$ ,  $\phi$  and  $\sigma$  unknown. Priors:

- ▶  $\mu \sim N(b_\mu, B_\mu)$
- ▶  $(\phi + 1)/2 \sim \mathcal{B}(a_0, b_0)$  as in Kim, Shephard, and Chib (1998), implying

$$p(\phi) = \frac{1}{2B(a_0, b_0)} \left(\frac{1+\phi}{2}\right)^{a_0-1} \left(\frac{1-\phi}{2}\right)^{b_0-1}$$

- ▶  $\sigma^2 \sim B_\sigma \cdot \chi_1^2 = \mathcal{G}\left(\frac{1}{2}, \frac{1}{2B_\sigma}\right)$
- ▶  $h_0 | \mu, \phi, \sigma \sim N\left(\mu, \sigma^2 / (1 - \phi^2)\right)$

R function Volatility(cal="yz") provides simulated volatility using OHLC high frequency approach.

$$\sigma^2 = \sigma_o^2 + k\sigma_c^2 + (1 - k)\sigma_{rs}^2$$

$$\sigma_o^2 = \frac{N}{n-1} \sum \left( \log \frac{O_i}{C_{i-1}} - \mu_o \right)^2$$

$$\mu_o = \frac{1}{n} \sum \log \frac{O_i}{C_{i-1}}$$

$$\sigma_c^2 = \frac{N}{n-1} \sum \left( \log \frac{C_i}{O_i} - \mu_c \right)^2$$

$$\mu_c = \frac{1}{n} \sum \log \frac{C_i}{O_i}$$

$$\sigma_{rs}^2 = \frac{N}{n} \sum \left( \log \frac{H_i}{C_i} \times \log \frac{H_i}{O_i} + \log \frac{L_i}{C_i} \times \log \frac{L_i}{O_i} \right)$$

$$k = \frac{\alpha - 1}{\alpha + \frac{n+1}{n-1}}$$

# Stochastic Interest Model

Ho Lee - a short interest rate model

Drift is adjusted to fit current term structure of interest rate

$$dr(t) = \sigma dW(t) + \mu(t)dt$$

$$\begin{aligned} r(t) &= r(0) + \sigma W(t) + \int_0^t \mu(s)ds \\ &= \sigma W(t) + b(t) \end{aligned}$$

$$r_t = r_{t-1} + \mu dt + \sigma \sqrt{(dt)} W_t$$

High Frequency Approach:

To infer the rate from high-frequency option prices

$$p - c = \alpha + \beta K + \epsilon$$

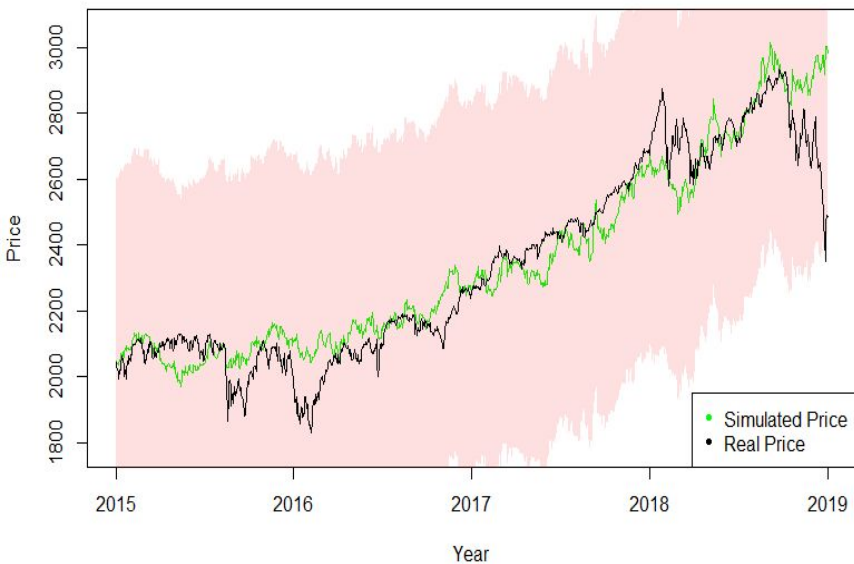
$$\beta = \exp(-r * T)$$

$$r = -\frac{1}{T} \log(\beta)$$

# Volatility Model Result

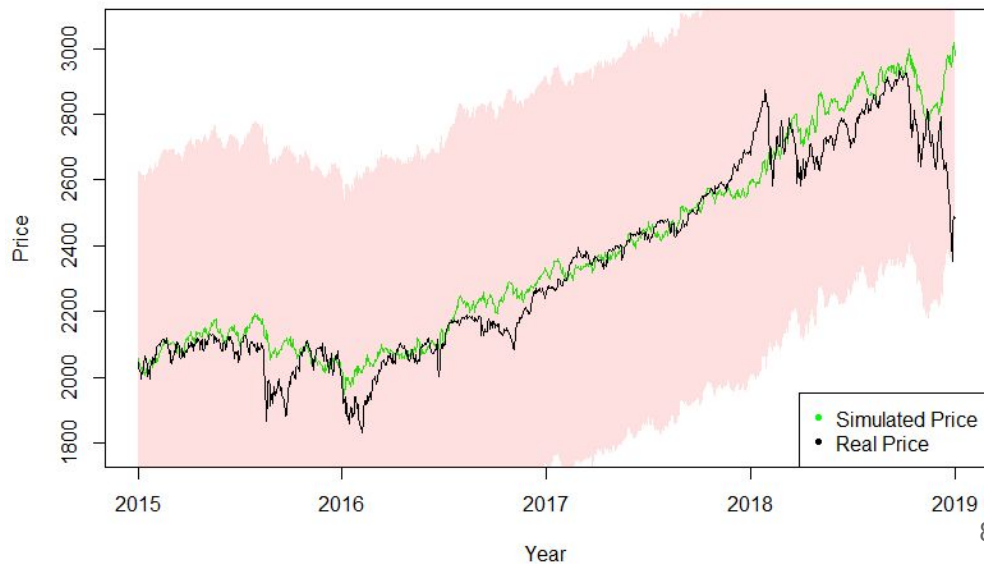
**Method: MCMC Stochastic Volatility**  
NRMSE: 0.04315852

**S&P500 Predicted Stock Price From 2015 to 2019**



**Method: High Frequency Approach (Yang & Zhang)**  
NRMSE: 0.03906292

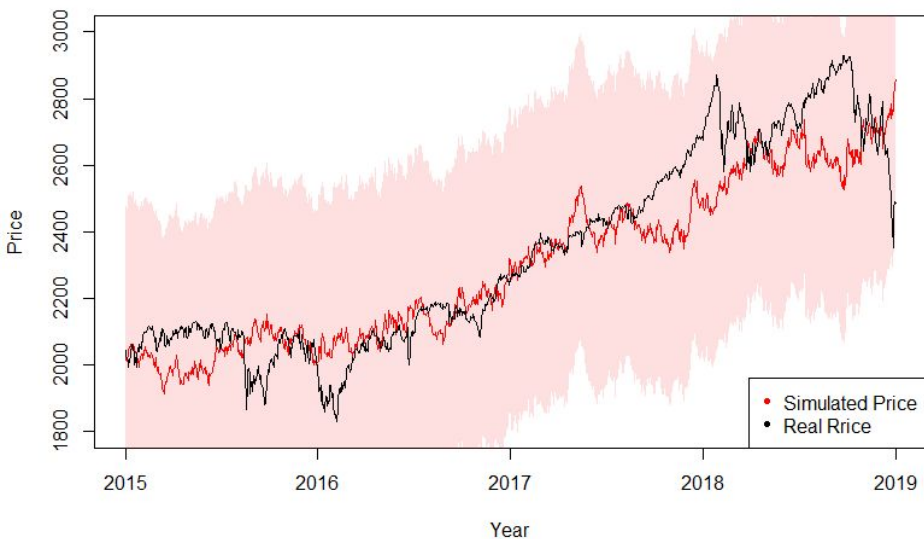
**S&P500 Predicted Stock Price From 2015 to 2019**



# Interest Model Result

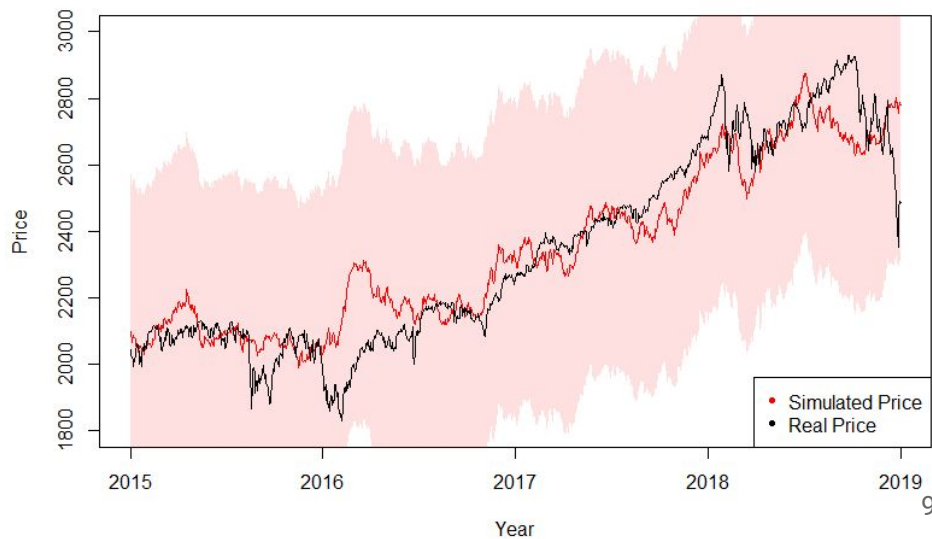
**Method: Ho Lee Interest Rate**  
NRMSE: 0.05464971

**S&P500 Predicted Stock Price From 2015 to 2019**



**Method: High Frequency Approach (2019)**  
NRMSE: 0.04586144

**S&P500 Predicted Stock Price From 2015 to 2019**



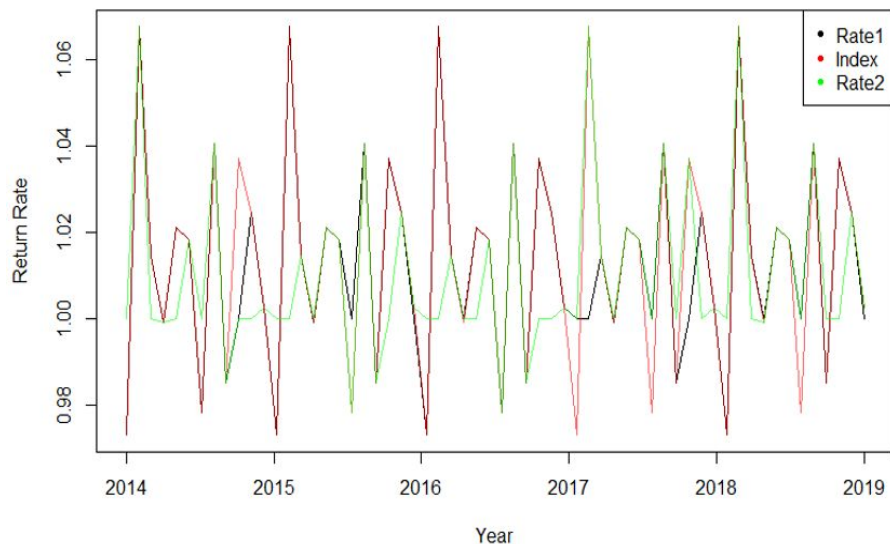
# Investment Strategy

- Predict index price: monthly returns
  - Based on past 60-month data
- Invest:
  - Simulated monthly return  $> 0.00\%$ . Invest in the beginning and close position in the end. Periodical index return as current period return;
  - Simulated monthly return  $\leq 0.00\%$ . Hold on the cash. Current period return as 1.
- Leverage: None vs. 5

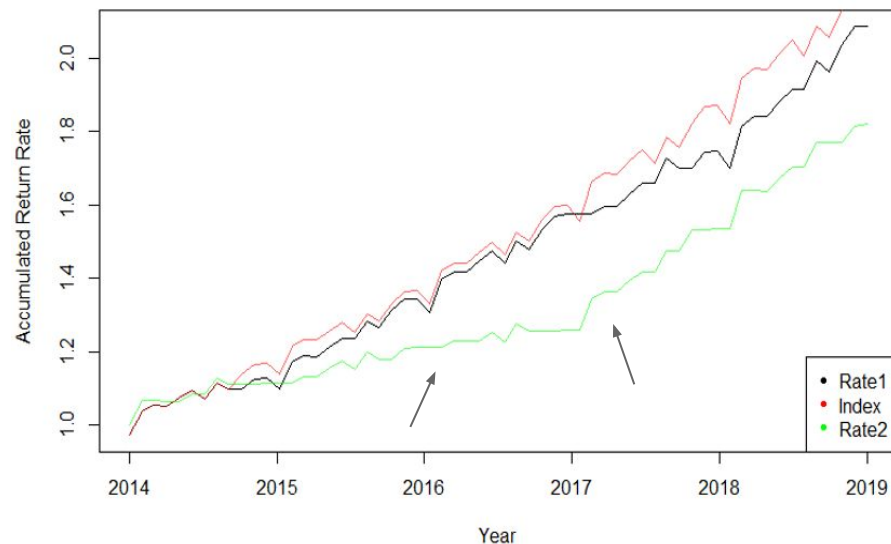


# Backtest Result: No Leverage

Investment Strategy Return Rate



Investment Strategy Accumulated Return Rate

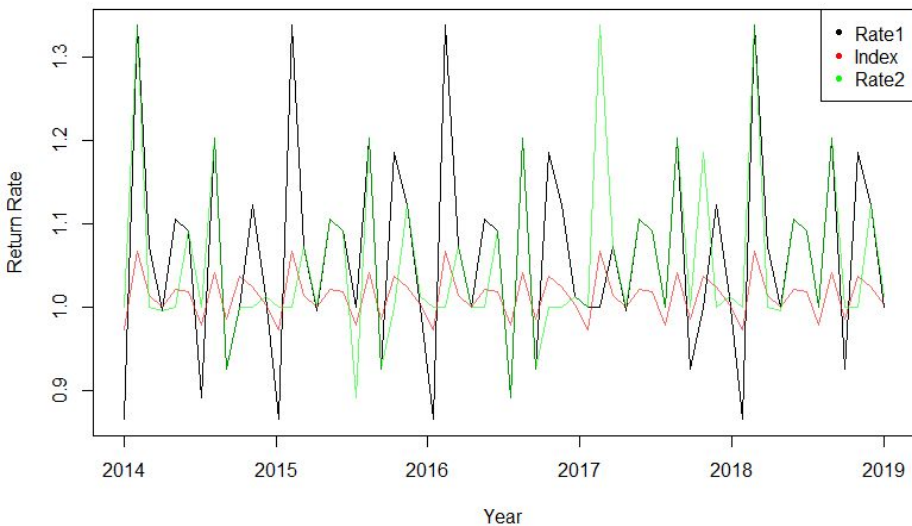


# Comments On No Leverage result

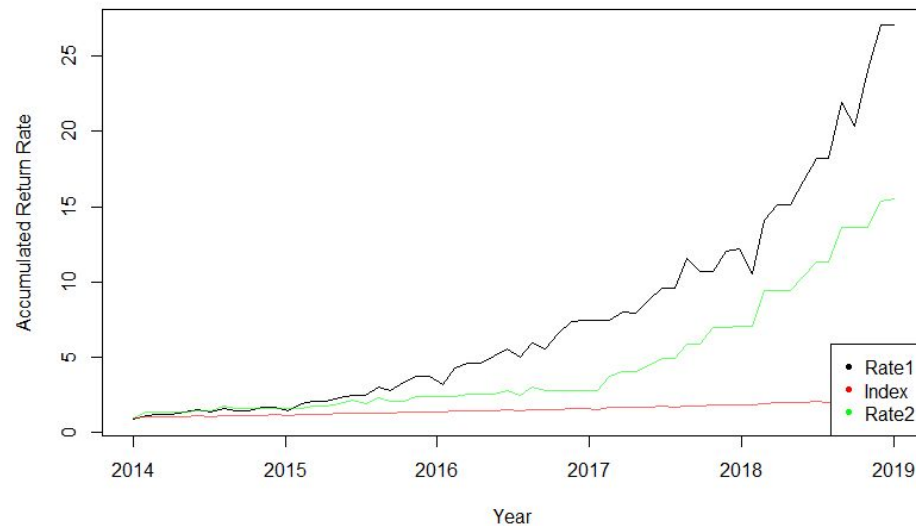
- Beat by SP500 benchmark return
  - SP500 index: 119.0%
  - Model 1: 108.6%, Model 2: 82.1%
- Reason:
  - Hypothesis: Model secure the directions in sacrifice of comprehensiveness
- Thus: Leverage 5

# Backtest result: Leverage 5

Investment Strategy Return Rate with 5 Leverage



Investment Strategy Accumulated Return Rate with 5 Leverage



# Comments On Leverage 5 result

- Result interpretation:
  - Return: Model 1: 2602.3%, Model 2: 1453.9%, SP500 119.0%
  - Justify the hypothesis: give up increasing to protect portfolio from loss
- Limitations:
  - High risk
  - May loss a lot during a market crash (eg. 2002, 2008 )
  - May only works during bull market
    - Blow up the market performance
    - need further test on bear market

# Prospective Research

- Dividends should be included
- Stock price isn't GBM in reality
- Negative interest rate and negative asset price should be considered
- More indices should be considered
- How to control the randomness
- Other fundamental factors might affect stock price

# Conclusion

- Model 1 > Model 2, both leverage 1 and 5
  - Interpretation: (Hypothesis)
    - Volatility has greater influence on the price
    - Volatility estimator (Zhang&Yang): minimum estimation error, independent of drift and opening gaps
  - Further investigation: seek causal effect and sensitivity analysis on interest rate and volatility
- Leverage 1: SP500 > M1 > M2; Leverage 5: M1 > M2 >> SP500

# Acknowledgement

- We would like to thank Professor Dobelman for the generous advice and support.

# References

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2. Yang, Dennis, and Qiang Zhang. "Drift Independent Volatility Estimation Based on High, Low, Open, and Close Prices." *The Journal of Business*, vol. 73, no. 3, 2000, pp. 477–492., doi:10.1086/209650.
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4. T.S.Y. Ho, S.B. Lee, *Term structure movements and pricing interest rate contingent claims*, *Journal of Finance* 41, 1986. doi:10.2307/2328161



Thank you!