STAT 280-Test 2 Solutions, Spring 2005

1) C $\quad P(x \geq .75) = P(z \geq \frac{.75-.35}{.33}) = P(z \geq \frac{.4}{.33}) = P(z \geq 1.2121) = .1127 \approx .113$

2) C $\quad P(x \geq 800) = P(z \geq \frac{800-336.25}{\sqrt{1456}}) = P(z \geq \frac{463.75}{38.16}) = P(z \geq 12.15) = Zeeeero$.

NOTE: $z \neq \frac{463.75}{1456} = .3185 \rightarrow p = 1-.3745$

3) C Basic definition.

4) D $\quad C = \bar{X} \pm \text{ME}; \quad \bar{X}$ is just the center of C

5) D $\quad P(x \leq 250) = P(z \leq \frac{250-445.5}{177.8}) = P(z \leq \frac{-195.5}{177.8}) = P(z \leq -1.1) = .136$ NOTE: 1-.136 is .86, one of the wrong answers.

6) C $\quad \mu \in C = \bar{X} \pm \text{ME}. \qquad \text{ME} = 10 = 1.645(\sigma/\sqrt{n}) = 65.8/\sqrt{n},\ \sqrt{n} = 6.58, n = 43.3 \nearrow 44$

7) The area under the normal curve represents probability and sums to 1.0. If we were to attempt to find the probability of a closing price exceeding \$4.50 based on the original normal distribution, we would have to integrate the normal distribution function from \$4.50 to infinity. Although possible, finding the integral of the normal distribution function is not a trivial matter. However, by converting the normal distribution to a standard normal distribution, we are determining a z value corresponding to the \$4.50 point. Then we can use the standard normal table to find the probability since the table contains integrals (areas) for all z values (to two decimal places) between 0.00 and 3.09.

8) A small sample impacts the estimation of a population mean in two main ways. First, in developing a confidence interval estimate for μ, we need the standard error of the sampling distribution. The standard error is computed as $\frac{\sigma}{\sqrt{n}}$. In a given situation, a small value of n will result in larger standard error. Then, when we develop the confidence interval using $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$, the width of the interval is greater than would be the case for a larger sample size. Thus, the margin of error is larger, which is undesirable. Further, in most applications, the population standard deviation, σ, is unknown and we must estimate it using s, the sample standard deviation. In these cases, when the sample size is small, we use the t-distribution to get the critical value for the confidence interval formula of the form: $\bar{x} \pm t\frac{s}{\sqrt{n}}$. Since the t-distribution is more spread out than the z-distribution, the width of the interval will be wider when the small sample size is used. Thus, the width is expanded in two ways-<u>larger standard error</u> and <u>larger critical value</u>.

9) A p-value is the probability of getting a sample mean that is as extreme or more extreme than the one observed from a population with the hypothesized parameter. For instance, if the sample mean that we find in our sample is "substantially" different than the hypothesized population mean, a small p-value will be computed. If the calculated p-value is less than alpha (α), the null hypothesis should be rejected.

10) The hypothesis test involves a test for the difference between population means and is based on large samples from each population. The appropriate null and alternative hypotheses are:

$$H_o : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$

This will be a <u>two-tailed test</u> since we are interested in testing whether there is a difference between the two population means with respect to miles per gallon and neither blend of gasoline is predicted to be superior to the other.

The critical value for a two-tailed test with a significance level of 0.05 is found in the Standard Normal table to be z = ±1.96. The test statistic in this case is computed using:

$$z = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(23.4 - 25.7) - 0}{\sqrt{\frac{16}{100} + \frac{17.64}{100}}} = \frac{-2.3}{.58} = -3.9655$$

Since <u>z = -3.9655 < -1.96</u>, we reject the null hypothesis and conclude that the population means are not equal. Based on the samples, <u>we infer that blend 2 will provide higher</u> mean mileage.

Although the question specifically said to "use the test statistic approach," one could also use the <u>confidence interval technique</u> to decide whether to reject the null hypothesis:
$C_{.95} = \Delta\mu_0 \pm 1.96 \cdot 0.58 = 0 \pm 1.1368$; since $\Delta\overline{X} = -2.3$ is outside *C*, we reject H_0.

11) The t-distribution is used to obtain the critical value for a confidence interval estimate for the population mean when the value for the population <u>standard deviation is unknown</u> and the sample size is reasonably small. Technically, the t-distribution can be used when the standard deviation is not known, but since the t-distribution and the z-distribution converge for large samples, it generally does not matter in cases where the sample size is large. <u>It should be noted</u> that the t-distribution is based on the assumption that the population is normally distributed. However, the t-distribution is usable as long as the population is "reasonably" symmetric.

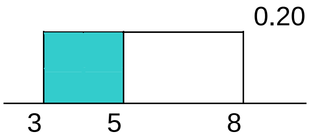
12) (<u>See diagram in answer to 14</u>). The continuous uniform probability distribution has a function:
f(x) = $\frac{1}{b-a}$ where *b* is the upper extreme of the distribution ($25.00) and *a* is the lower extreme ($5.00). Then f(x) = $\frac{1}{25.00 - 5.00} = \frac{1}{20}$ = 0.05. Now, p(8.00 ≤ x ≤ 12.00) = f(x)(12.00 - 8.00) = 0.05(4.00) = 0 .20. Thus, there is a 0.20 probability that someone will spend between $8.00 and $12.00 after getting into the amusement park. The chance that the second person will spend more than $15.00 is found as 0.05(25.00 - 15.00) = 0.50. Then using the multiplication rule for independent events, we find the desired probability as: 0.20 × 0.50 = 0 .10. Thus, there is a 0.10 chance that the event of interest will occur.

13) We are interested in finding P(p > 0.14). The sampling distribution for a proportion will be approximately normal as long as both nπ and n(1 - π) are greater than 5. That applies in this case. The standard deviation for the sampling distribution is given by $\sqrt{\pi(1-\pi)/n}$. Thus, to find the probability, we standardize the sample proportion as follows:

$$z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} = \frac{.14 - .10}{\sqrt{\frac{.10(.90)}{100}}} = \frac{.04}{.03} = 1.33$$

Then we can go to the standard normal table for z = 1.33. We get 0.0918, which is the probability we are looking for.

14) TRUE



$P(X \le 5) = 2(.2) = .4$

15) FALSE Mean is μ, $\sigma = 12/\sqrt{25}$

16) FALSE Confidence level $=(1-\alpha)$, does not depend on n or σ.

17) TRUE As you recall, $ME = z_{\frac{\alpha}{2}}\sqrt{pq/n}$; $n = pq\left(z/ME\right)^2$

18) TRUE

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{p_0 q_0\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}};\ p_0 = 21/200 = .105;$$

$$z = .05/\sqrt{.105(.895)(1/50)} = .05/\sqrt{.00188} = 1.153$$

19) TRUE

20) TRUE ME $= \pm z_{\frac{\alpha}{2}}(\sigma/\sqrt{300}) = \pm 1.96(\sigma/\sqrt{300})$, $\sigma = 3\sqrt{300}/1.96 = 26.51$