Joint Probability Distribution

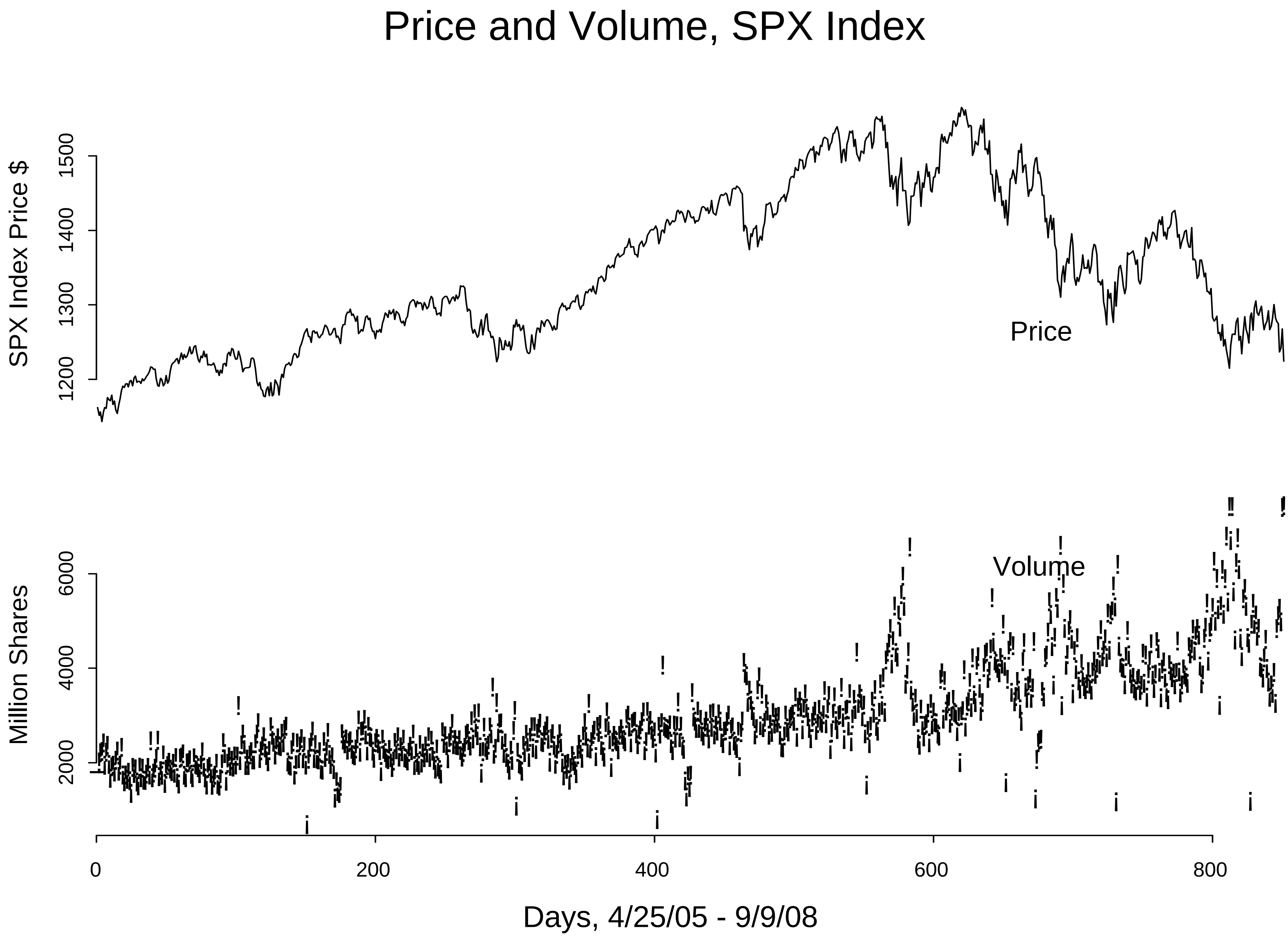
$$P \equiv P_t = P(t)$$

$$\left\langle P(t); t \in D \subseteq \mathbb{R}^d \right\rangle \quad t \in \mathbb{R}^1, t \geq 0$$

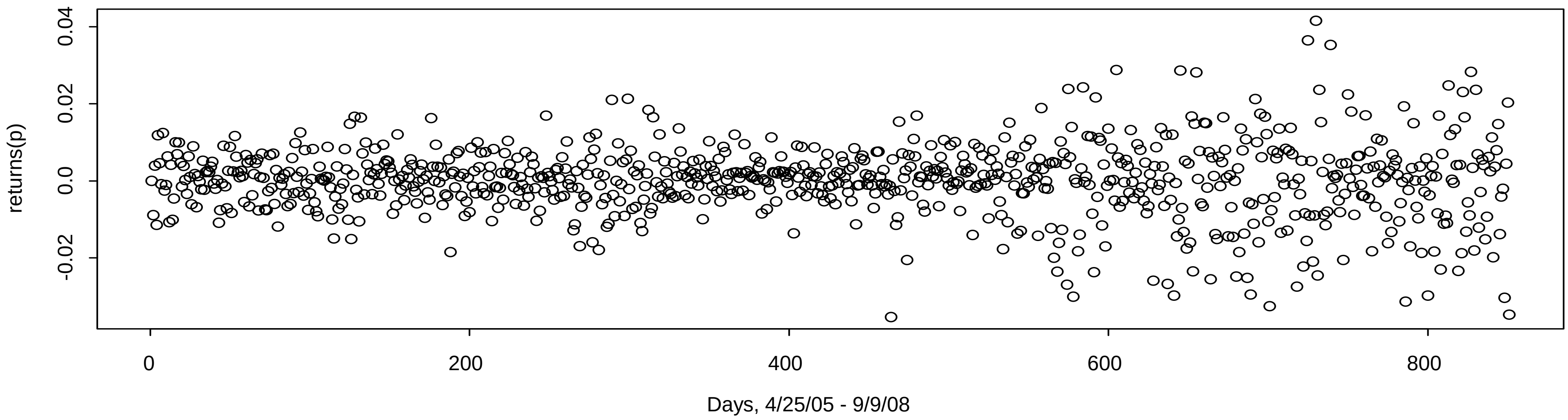
$$R_t = \frac{P_t}{P_{t-1}}$$

$$r_t = \ln(R_t) = \ln(r_{pct} + 1)$$

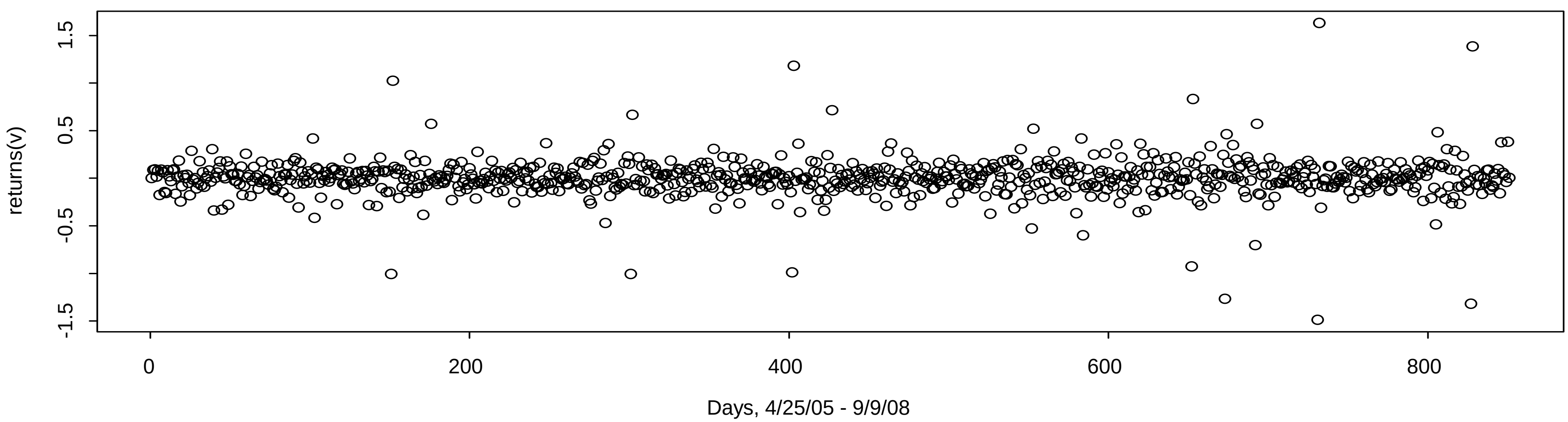
$$GBM(\hat{\mu}_{gbm}, \hat{\sigma}_{gbm})$$

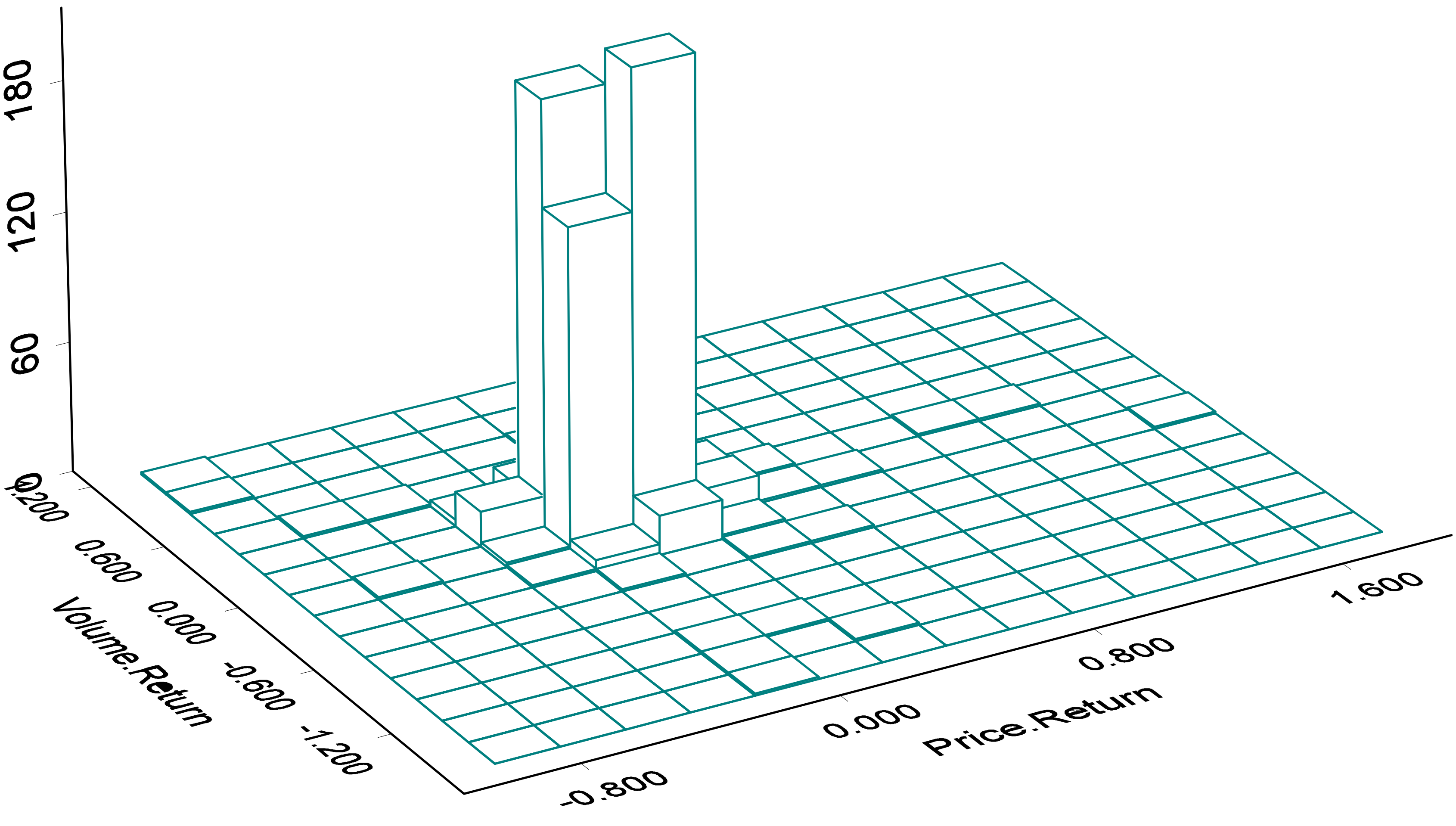

Price and Volume, SPX Index
SPX Index Price $
1200
1300
1400
1500
Price
Million Shares
2000
4000
6000
Volume
0
200
400
600
800
Days, 4/25/05 - 9/9/08

Returns, Price



Returns, Volume





180
120
60
0
1.200
0.600
0.000
-0.600
-1.200
Volume.Return
-0.800
0.000
0.800
1.600
Price.Return

- Consider a mixture distribution of Normal and Poisson shocks.

$$X \sim \text{Poisson}(\lambda): e^{-\lambda}\frac{\lambda^x}{x!};\ M_X(t) = e^{\lambda(e^t - 1)}$$

$$Y \sim N(\mu,\sigma^2): \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2};\ M_Y(t) = e^{t\mu + \frac{\sigma^2}{2}t^2}$$

$$M_X(t)M_Y(t) = e^{\lambda e^t + t(\mu + \sigma^2/2\, t) - \lambda}$$

Invert for pdf