$$\sqrt{n}\left(\hat{\theta}-\theta\right)\xrightarrow{d} N\left(0, I_1(\theta)^{-1}\right)$$

"Asymptotic normality of the mle (equation 15) is arguably the most important equation in theoretical statistics. It is really remarkable. We may have no formula that gives the MLE as a function of the data. We may have no procedure for obtaining the MLE except to hand any particular data vector to a computer program that somehow maximizes the log likelihood. Nevertheless, theory gives us a large sample approximation (and a rather simple large sample approximation) to the sampling distribution of this estimator, an estimator that we can't explicitly describe! Despite its amazing properties, we must admit two issues.

"First, we haven't actually proved it, and even if we wanted to make this course a lot more mathematical than it should be we couldn't prove it in complete generality. We could prove it under some conditions, but those conditions don't always hold. Sometimes (15) holds and sometimes it doesn't. There must be a thousand different proofs under various conditions in the literature. It has received more theoretical attention than any other result. But none of those proofs apply to all applications.

"And even if we could prove it to hold under all conditions (that's impossible, because there are counterexamples, applications where it doesn't hold, but assume we could), it still wouldn't tell us what we really want to know. It only asserts that for some sufficiently large n, perhaps much larger than the actual n of our actual data, the asymptotic approximation of normality would be good. But perhaps it is no good at the actual n where we want to use it. Thus (15) has only heuristic value. The theorem gives no way to tell whether the approximation is good or bad at any particular n. Of course, now that we know all about the parametric bootstrap that shouldn't bother us. If we are worried about whether (15) is a good approximation, we simulate. Theory is no help. " -Charles Geyer, UMN July 13, 2013

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