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Statistics 100A | **Instructor: Nicolas Christou**

The central limit theorem
The distribution of the sample mean
The distribution of the sum

Suppose a population has mean μ and standard deviation σ. Let $X_1, X_2, \cdots, X_n$ be an i.i.d. (independent and identically distributed) sample from this population. This means that $E(X_i) = \mu$, and $Var(X_i) = \sigma^2$. Define the following random variables:

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}.$$

and

$$T = X_1 + X_2 + \cdots + X_n = \sum_{i=1}^{n} X_i.$$

Then, for large n (usually $n > 30$) the following statements are approximately true regardless of the shape of the population:

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

therefore

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

and

$$T \sim N(n\mu, \sigma\sqrt{n})$$

therefore,

$$Z = \frac{T - n\mu}{\sigma\sqrt{n}}$$

Note: If the population from where the sample is selected follows the normal distribution, the above statements are true regardless of the sample size. In this case n can be small or large.

How can we use the above statement? Here is an example:
A large freight elevator can transport a maximum of 9800 pounds. Suppose a load of cargo containing 49 boxes must be transported via the elevator. Experience has shown that the weight of boxes of this type of cargo follows a distribution with mean $\mu = 205$ pounds and standard deviation $\sigma = 15$ pounds. Based on this information, what is the probability that all 49 boxes can be safely loaded onto the freight elevator and transported?