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Empirical properties of asset returns: stylized facts and statistical issues

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Abstract

We present a set of stylized empirical facts emerging from the statistical analysis of price variations in various types of financial markets. We first discuss some general issues common to all statistical studies of financial time series. Various statistical properties of asset returns are then described: distributional properties, tail properties and extreme fluctuations, pathwise regularity, linear and nonlinear dependence of returns in time and across stocks. Our description emphasizes properties common to a wide variety of markets and instruments. We then show how these statistical properties invalidate many of the common statistical approaches used to study financial data sets and examine some of the statistical problems encountered in each case.

Although statistical properties of prices of stocks and commodities and market indexes have been studied using data from various markets and instruments for more than half a century, the availability of large data sets of high-frequency price series and the application of computer-intensive methods for analysing their properties have opened new horizons to researchers in empirical finance in the last decade and have contributed to the consolidation of a data-based approach in financial modelling.

The study of these new data sets has led to the settlement of some old disputes regarding the nature of the data but has also generated new challenges. Not the least of them is to be able to capture in a synthetic and meaningful fashion the information and properties contained in this huge amount of data. A set of properties, common across many instruments, markets and time periods, has been observed by independent studies and classified as 'stylized facts'. We present here a pedagogical overview of these stylized facts. With respect to previous reviews [10, 14, 16, 50, 95, 102, 109] on the same subject, the aim of the present paper is to focus more on the properties of empirical data than on those of statistical models and introduce the reader to some new insights provided by methods based on statistical techniques recently applied in empirical finance.

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Our goal is to 'let the data speak for themselves' as much as possible. In terms of statistical methods, this is achieved by using so-called *non-parametric* methods which make only qualitative assumptions about the properties of the stochastic process generating the data: they do not assume that they belong to any prespecified parametric family.

Although non-parametric methods have the great theoretical advantage of being model free, they can only provide qualitative information about financial time series and in order to obtain a more precise description we will sometimes resort to semi-parametric methods which, without completely specifying the form of the price process, imply the existence of a parameter which describes a property of the process (for example the tail behaviour of the marginal distribution).

Before proceeding further, let us fix some notations. In the following, $S(t)$ will denote the price of a financial asset—a stock, an exchange rate or a market index—and $X(t) = \ln S(t)$ its logarithm. Given a *time scale* Δt, which can range from a few seconds to a month, the log return at scale Δt is defined as:

$$r(t, \Delta t) = X(t + \Delta t) - X(t). \tag{1}$$

In many econometric studies, Δt is set implicitly equal to one in appropriate units, but we will conserve all along the variable Δt to stress the fact the properties of the returns depend

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