Chapter 3

3.0 Parameter Optimization for the S&P 500.

This chapter presents unique problems and results for $\Omega^0 = \Omega^{500}$, the stocks comprising the SP-500. It also provides the framework and approach needed to extend this research into Ω^{5000}. We first present some baseline results from the single-group optimization, and then discuss various portfolio separation concepts, and conclude with some performance issues.

3.1 Single Group Results

The natural way to proceed after obtaining the SP-100 results was to repeat the process for the SP-500 universe, except with a dimensionality on the order of 500, where there was no assurance N-M would converge at all. The amazing thing was that it worked, but it took a long time, as we showed in the last section of chapter 2. The first baseline results used the same CS0 parameter settings as the initial SP-100 trials. These are:

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SP-500 Constraint Set
=====================
optimization method -----> nonparametric, One group
portfolio size -----> various, 380-498
projected distribution time (years) -----> 1
initialization vector ----->  Equal-wt
maximization criteria -----> Median
minimum risk percentile -----> 20
minimum risk return value -----> 1.05
number of simulations -----> 1000
maximum allocation -----> .05
```

For comparison purposes, since Compustat's SP-500 membership flags only traced back to 1976, we will be restricting our SP-100 comparisons to the 26-year period from 1977 through 2002. We could back-estimate prior results based on geomean returns, but it is more fair to just move up the SP-100 start dates until 1977.

Table 3.1. SP-500 Baseline Results and performance comparison,, constraint set 0. SP-500 constraint set: optimization method: nonparametric, One group; portfolio size: 380 - 498; projected distribution time (years): 1; initialization: mktcap; maximization criteria: median; simulations: 1000; minimum risk percentile: 20; minimum risk return: 1.05; max allocation: .05.

# Stocks / year			Mkt Cap Weighted		Equal Wt.		SP 100	SP 500	
100	500	Year	SP-100	SP-500	SP-100	SP-500	Simugram	1 sample	Δ
75	397	1977	-0.110	-0.121	-0.055	-0.065	0.0008	0.0898	0.09
75	396	1978	0.047	0.040	0.095	0.067	0.1230	0.1169	-0.01
75	396	1979	-0.009	0.114	0.161	0.242	0.3854	0.2512	-0.13
77	395	1980	0.216	0.302	0.318	0.282	0.6158	0.6625	0.05
77	392	1981	-0.087	-0.117	-0.035	-0.014	-0.1083	-0.1857	-0.08
77	394	1982	0.245	0.144	0.329	0.235	0.4861	0.3898	-0.10
76	395	1983	0.183	0.203	0.252	0.280	0.3617	0.3106	-0.05
78	396	1984	0.028	0.012	0.004	-0.022	-0.0150	-0.0846	-0.07
79	395	1985	0.269	0.264	0.343	0.261	0.3619	0.3035	-0.06
80	387	1986	0.129	0.149	0.182	0.143	0.2933	0.2766	-0.02
85	396	1987	0.014	0.041	0.024	0.050	0.1517	0.0898	-0.06
85	396	1988	0.064	0.081	0.093	0.123	0.0021	-0.0014	0.00
87	388	1989	0.279	0.254	0.323	0.216	0.3771	0.2985	-0.08
87	386	1990	-0.036	-0.059	-0.069	-0.143	0.0764	-0.1371	-0.21
87	383	1991	0.357	0.290	0.461	0.326	0.6909	0.6074	-0.08
90	382	1992	0.049	0.032	0.198	0.115	0.3218	0.0547	-0.27
92	383	1993	0.051	0.062	0.177	0.128	0.4944	0.4184	-0.08
95	380	1994	0.018	0.015	0.022	0.015	0.0541	-0.0089	-0.06
96	496	1995	0.395	0.337	0.435	0.283	0.5118	0.2697	-0.24
97	494	1996	0.253	0.195	0.239	0.158	0.3333	0.2622	-0.07
97	494	1997	0.352	0.315	0.386	0.270	0.5146	0.3142	-0.20
99	498	1998	0.357	0.261	0.330	0.112	0.7678	0.4538	-0.31
99	494	1999	0.276	0.173	0.291	0.109	0.7341	0.7722	0.04
99	498	2000	-0.095	-0.065	0.073	0.111	-0.1392	-0.3099	-0.17
99	495	2001	-0.104	-0.099	-0.067	0.019	-0.1427	-0.1962	-0.05
99	496	2002	-0.245	-0.236	-0.212	-0.194	-0.2499	-0.2208	0.03
Terminal $ Value			11.3	9.0	39.3	15.4	245.8	39.0	0.085
mean %			11.1%	9.9%	16.5%	12.0%	26.9%	18.5%	-8.5%
		$\hat{\sigma}$	18%	16%	18%	14%	29%	28%	-8.5%
geomean % (annualized)			9.8%	8.8%	15.2%	11.1%	23.6%	15.1%	-9.0%

There are several things to note about this first baseline result. To save space, a market benchmark is not specifically included; we can use the two MW portfolios on the left.

When we do this we are reminded of the difference between the Wilshire 5000 and the rest of the indexes, the former outperformed by about the same amount. This is the first indication that there is something different about the SP-100 vs. the SP-500, a theme that will recur and is not really resolved at the time of this writing.

In the EW returns, we again see the unusual characteristics discussed in previous chapters. Even the SP-500 EW returns are 20-25% greater than the MW versions. We see annual volatility numbers for these proxy indexes in line with longer-term estimates for the real indexes, except for the lower volatility on the SP-500 EW.

The SP-100 simugram is the 105-sample CS0 version already discussed, which has become the new benchmark for the SP-100 simugram, and next to it is the SP-500 simugram column. We note its performance is substantially better than either of the two MW proxy indexes. But, when one is accustomed to the returns that chapters 1 and 2 provided, the joy is tempered. Indeed, the annualized return is almost twice that of the SP-500, and 50% that of the Wilshire 5000 proxy, with a respectable TV 4 times that of "the market." But forgetting the simugrams, the EW Wilshire gives $106M TV, and 19.6% geomean return over this 26-year period (recall the latter returned $157M and 16.6% for 33 years). It was for this reason that so much work went into validating the SP-100 simugram technique.

One wonders how the bottom lines could be so far apart when it appears that the returns are outstanding for any given year. The SP-500 simugram outperformed its own SP-500 benchmark 19 out of 26 years, or 73% of the time. But, it matched or underperformed the SP-100 simugram in 11/13 years, or 85% of the time. The "Δ" column tracks the SP-500 outperformance. Its mean value is –8.5%. An investment with those return loses 91.5% in 26 years. When one graphs the cumulative outperformance, a disappointing profile results.

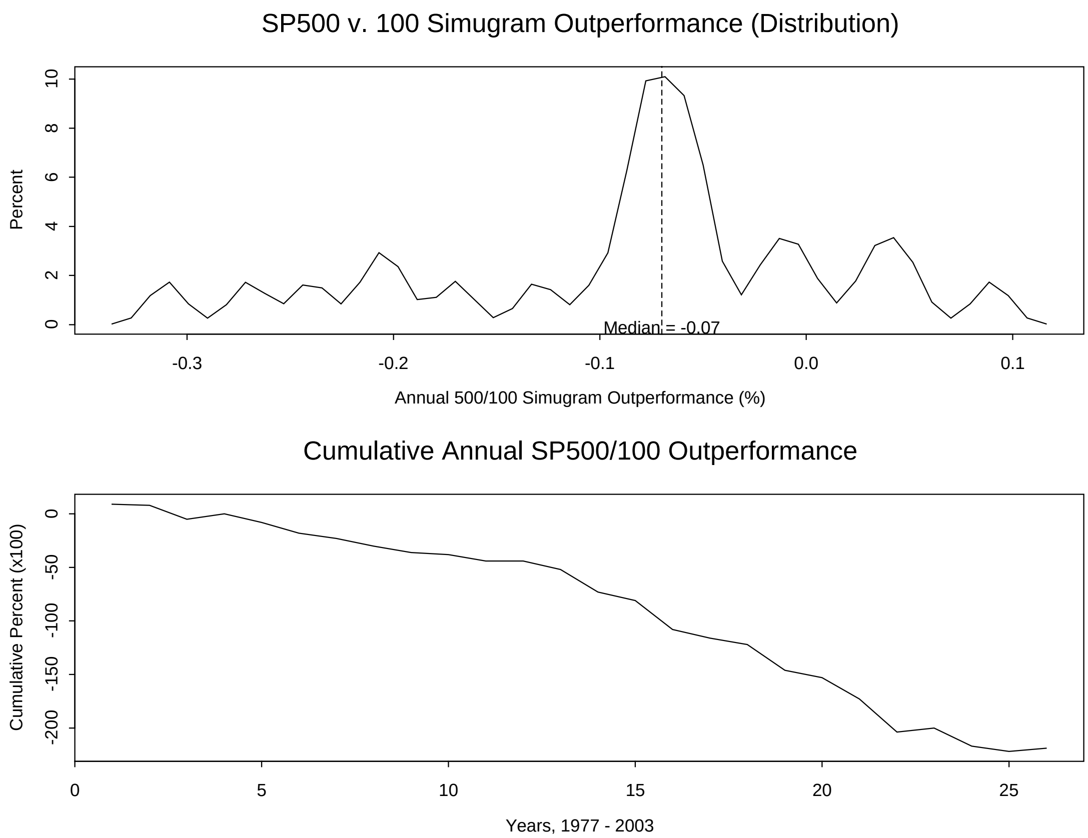

Figure 3.1 Cumulative performance of SP-500 vs. SP-100 simugram

It appeared that either (i) the SP-100 results were wrong; (ii) the N-M algorithm was unable to escape local maxima in this high dimensional space and hence stalling; or (iii) there is a fundamental reason why stocks in this index will contribute to simugram outperformance when the traditional methods have succumbed to EMH normal returns.

Option (i) was sufficiently studied and found to be incorrect, i.e., there is no mistake, non-causality, or other pathology in the approach or code. Option (iii) could in fact be in operation, but that is the topic of another dissertation. This left option (ii) as the culprit, and several remedial measures were available.

A simple solution would be if the problem were caused by a bad starting place. Since the first runs were made with initialization weights that were equal weighted, i.e., $1/K$, then perhaps market capitalization initial weights would have a better result. If not, then some sort of portfolio splitting technique could be employed. If not that, then a procedure such as simulated annealing could be employed. If that was ineffective, then another optimzer could be used. The first two of these techniques were used and comprise the bulk of this chapter.

If the MC initial weights did not make a difference, then that means random weights would not make a difference either. The MC weights were coded and several simulations run on the SP-100. These results in table 3.2 are typical of many of the runs. Based on 10 samples for both equal- and market cap- initial values, the 33-year cumulative sum of outperformance of MC over EW is –0.02. The average range of standard deviation for these differences is 0.6, so there is no difference.

Table 3.2 Mean simugram returns by initial weight vector, EW vs. MC. Sample size is 11 for the market-cap initial weights, and 105 for the equal-weight initialization vector.

Mean Simugram Returns by X_0

Year	Eq_wt Initial X	Mkt_wt Initial X	Differences M - E Diff	Differences M - E times σ
1970	-0.110	-0.102	0.008	0.925
1971	0.329	0.328	-0.001	0.033
1972	0.417	0.418	0.001	0.074
1973	-0.069	-0.071	-0.003	0.330
1974	-0.311	-0.313	-0.001	0.228
1975	0.375	0.357	-0.018	0.330
1976	0.322	0.321	-0.001	0.067
1977	0.001	0.013	0.012	0.594
1978	0.123	0.121	-0.002	0.291
1979	0.386	0.382	-0.004	0.311
1980	0.616	0.620	0.004	0.244
1981	-0.109	-0.106	0.002	0.113
1982	0.486	0.486	0.000	0.036
1983	0.362	0.360	-0.002	0.146
1984	-0.016	-0.012	0.003	0.238
1985	0.362	0.373	0.011	0.587
1986	0.294	0.288	-0.006	0.461
1987	0.151	0.152	0.000	0.063
1988	0.003	0.006	0.003	0.209
1989	0.378	0.373	-0.005	0.310
1990	0.077	0.081	0.004	0.208
1991	0.692	0.683	-0.009	0.336
1992	0.323	0.321	-0.002	0.062
1993	0.495	0.488	-0.007	0.683
1994	0.054	0.055	0.001	0.090
1995	0.512	0.519	0.008	0.318
1996	0.333	0.331	-0.002	0.099
1997	0.514	0.510	-0.004	0.209
1998	0.766	0.777	0.011	0.324
1999	0.733	0.729	-0.005	0.150
2000	-0.140	-0.137	0.004	0.327
2001	-0.143	-0.151	-0.008	1.005
2002	-0.249	-0.243	0.006	0.254
Terminal $ Value	480.5	484.8		
mean %	24.1%	24.1%		29.3%
geomean % (annualized)	20.6%	20.6%		

During the sampling tests described in section 2.6, 500 trials for both equal- and market cap-weight were conducted, and show the indistinguishibility of the two techniques:

1982	N=150	
cs25; K=77		
M=1000		
	Eq Wt	Mkt Wt
Mean	0.651	0.640
Stddev	0.127	0.134
CoefVar	0.195	0.209
Mktwt/Eqwt - 1:		-1.6%

1999	N=350	
cs30; K=99		
M=1000		
	Eq Wt	Mkt Wt
Mean	1.011	0.997
Stddev	0.199	0.187
CoefVar	0.197	0.187
Mktwt/Eqwt - 1:		-1.4%

The slight average negative difference is borne out in visual inspection of histograms if this difference, leading one to the tentative finding that using initialization based on the market-cap weights may lead to slightly smaller returns; however, these and later comparisons with earlier SP-500 26-year baseline runs leads us to the conclusion that there is no significant difference in returns based on the starting vector. The possible smaller returns should be quantified more fully.

3.2 Portfolio Splitting

This left the stalling problem, which would most easily be solved by splitting the portfolio into other smaller ones and combining the results. This portfolio splitting technique (PST) was accomplished in three phases. First, a split was made into two random groups of about 250 stocks each. Then, three groups were split; and finally, a 2-pass operation was implemented whereby the portfolio was split into 5 groups of 100 stocks each, the best simugram weights applied to all, keeping those in each group’s viable security list (VSL). We then re-optimized on that reduced portfolio. It was well established that the routine could handle 100 stocks, and this would pave the way for handling larger portfolios than the SP-500. In the development process the technique was described as PST, but in reality it was dubbed DAC (divide and conquer), so that term may appear in this dissertation.

We must first review some basic facts on multiple portfolio returns. When one has capital C and decides to split it into k portfolios, say $k = 2$, then she plans on allocating her capital as $C_1 = C_2 = \frac{C}{2}$, and investing $I_j = C_j$ in P_j. However, this is only true under the implicit assumption that the weights in each portfolio add to 1. In general the weights would add to $\sum_i w_{ij}$. That is, $I_j = \Sigma w_j C_j$. If each portfolio earns r_j, then the dollar return is $R_j = r_j I_j$. For a stock portfolio, the return of the jth portfolio can be calculated as $r_j = \frac{\Sigma \Delta I}{\Sigma I} = \frac{C_j \Sigma w_i r_{X_i}}{C_j \Sigma w_i}$. So IF $I_1 = I_2 = \frac{C}{k}$ and IF $\Sigma w_1 = \Sigma w_2 = 1$, then the total return simplifies to $r = \frac{r_1 + r_2 + \cdots r_k}{k} = \overline{r_i}$. But this is only true if the sum of the weights are equal, and equal to 1. If the Σw are equal but equal to $1/k$, then the investment $I_j = \Sigma w_j C_j = \frac{C_j}{k}$. With $k = 2$, $C_i = \frac{C}{2}$, but $I_1 = \frac{C}{4}$, only half of what was intended for investment. This resolves so long as the allocations are equal, and the combined portfolio return works out to $r = \frac{kr_1 + kr_2 + \cdots + kr_k}{k} = k\overline{r} = \Sigma r_i$.

This exercise is only necessary because in reality we do not want $\Sigma w = 1$ in each portfolio. Since the simugram assigns viable weights to $1/a + n_a$ symbols, with an allocation of 5% we would have 30 stocks with more or less equal allocation in each portfolio, for a total of 60. This might be fine for diversification, but it also implies there are twice as many "not so good" stocks in total. With 3 portfolios there are 90 stocks with equal allocation, etc. These portfolios generate simugram returns, but it seems they can hardly outperform an EW index. These groups are referred to the DAC-k Sum k, with sum of weights equal to k, the number of groups.

What we really want is k groups where the weights sum to 1 so that Σw in each group = 1/*k*. This ensures the resulting portfolio has $1/a+n_a$ symbols as intended. This is referred to as DAC-k Sum 1, with sum of weights equal to 1. There is a problem with executing the simugram system when the weights sum to $1/k$. If the minimum tail return is not scaled properly, then it will be impossible to obtain the unscaled minimum return with the stocks at hand, and the weight constraints will be violated, corrupting the sub-portfolio return. It just can't be done with the good stocks only being able to sum to $1/k$. There are two major implications to this twist. The first is that new r* values must be determined for each year for each of k groups. Depending on how the groups are selected, these values may be totally different. The second implication is for a minimum group size of 100 say, for a total of 5 groups, then the sum of the weights would only sum to .20, resulting in a silly value for r*. Since one of our objectives was to develop a means to work with very large collection of stocks, this did not seem viable.

However, if 2-pass optimization were used, then we could split the stocks into k groups with the sum of weights of each equal to 1, not worrying about blowing the weight constraints since we were only interested in the VSL from each group. This was the essence of the 2-pass optimization approach, which will work on any size portfolio.

The feeling was that the DAC Sum 1 would perform better than DAC sum k. We pose this in the following manner. The usual alternative portfolio and universe is $P^A = (\Omega^0, w_A)$, and we split it into k groups such that $P^A = \bigcup_{i=1,k} P_i,\ P_i \bigcap P_j = \varnothing$. In our business strategy S set there are several regarding our portfolios, denoted $S_P \in S$. The sum of the weights in P^A will all be the same, $\sum_{j=i}^{k} w_i = c_j = \begin{cases} 1 \\ 1/k \end{cases}$, according to the weight sum strategy, $s1, sk \in S_P$. Given this, we state Hypothesis 3.

$$H_0 : \quad \left\| r(P_{s1}{}^A) - r(P_{sk}{}^A) \right\|_T = 0$$
$$H_3 : \quad \left\| r(P_{s1}{}^A) - r(P_{sk}{}^A) \right\|_T > 0$$

Prior to implementing the 2-pass optimization, tests for this had be conducted.

3.2.1 2-Group Portfolio (DAC-2)

The first attempt to thwart stalling was to split into $k = 2$ groups, with Σw=2, and the results are presented in table 3.3 below.

Table 3.3 SP-500 simugram, DAC-2, sum(w)=2 SP-500 constraint set: optimization method: nonparametric, 2 groups; portfolio size: 380 - 498; projected distribution time (years): 1; initialization: mktcap; maximization criteria: median; simulations: 1000; minimum risk percentile: 20; minimum risk return: 1.05; max allocation: .05.

N	Year	Wil 5000	Geomkt	DAC-2 Σ(w)=2 Sum gram	Samps
397	1977	-0.070	-0.120	0.0499	2
396	1978	0.040	0.006	0.2187	2
396	1979	0.193	0.118	0.3175	2
395	1980	0.522	0.226	0.6366	12
392	1981	-0.084	-0.091	-0.1646	3
394	1982	0.129	0.157	0.3895	2
395	1983	0.187	0.188	0.2660	3
396	1984	-0.013	-0.012	-0.0919	2
395	1985	0.272	0.271	0.3547	2
387	1986	0.125	0.165	0.2364	2
396	1987	-0.007	0.012	0.0552	2
396	1988	0.133	0.125	0.0050	2
388	1989	0.249	0.264	0.3369	2
386	1990	-0.093	-0.068	-0.0958	2
383	1991	0.303	0.256	0.4709	2
382	1992	0.062	0.049	0.1293	2
383	1993	0.086	0.097	0.3771	3
380	1994	-0.025	-0.007	0.0347	2
496	1995	0.334	0.336	0.3477	2
494	1996	0.188	0.217	0.2692	2
494	1997	0.292	0.276	0.2504	2
498	1998	0.217	0.214	0.3421	2
494	1999	0.220	0.222	0.5342	1
498	2000	-0.118	-0.094	-0.2512	3
495	2001	-0.121	-0.108	-0.1694	2
496	2002	-0.221	-0.208	-0.1515	2

Terminal $ Value		10.4	8.4	43
mean %		10.8%	9.6%	18.1%
	$\hat{\sigma}$	17%	15%	24%
geomean % (annualized)		9.4%	8.5%	15.6%

It is not necessarily easy to see if this is significantly different than the SP-500 1-group baseline. We performed a comparison with the CS25 and obtained the results below.

Table 3.4. SP-500 simugram, CS25, DAC-2, sum(w)=2. SP-500 constraint set: optimization method: nonparametric, One group; portfolio size: 380 - 498; projected distribution time (years): 1; initialization: mktcap; maximization criteria: median; simulations: 1000; minimum risk percentile: 20; minimum risk return: 1.05; max allocation: .05.

N	Year	Wil 5000	Geomkt	DAC-2 Σ(w)=2 cs0	DAC-2 Σ(w)=2 cs25
397	1977	-0.070	-0.120	0.0499	0.1154
396	1978	0.040	0.006	0.2187	0.1766
396	1979	0.193	0.118	0.3175	0.1491
395	1980	0.522	0.226	0.6366	0.8245
392	1981	-0.084	-0.091	-0.1646	-0.1776
394	1982	0.129	0.157	0.3895	-0.1186
395	1983	0.187	0.188	0.2660	0.3464
396	1984	-0.013	-0.012	-0.0919	-0.2007
395	1985	0.272	0.271	0.3547	0.5319
387	1986	0.125	0.165	0.2364	0.3719
396	1987	-0.007	0.012	0.0552	0.1404
396	1988	0.133	0.125	0.0050	0.0775
388	1989	0.249	0.264	0.3369	0.2193
386	1990	-0.093	-0.068	-0.0958	-0.1131
383	1991	0.303	0.256	0.4709	0.7787
382	1992	0.062	0.049	0.1293	-0.1186
383	1993	0.086	0.097	0.3771	0.7768
380	1994	-0.025	-0.007	0.0347	-0.0308
496	1995	0.334	0.336	0.3477	0.2715
494	1996	0.188	0.217	0.2692	0.3236
494	1997	0.292	0.276	0.2504	0.4180
498	1998	0.217	0.214	0.3421	0.4638
494	1999	0.220	0.222	0.5342	0.7760
498	2000	-0.118	-0.094	-0.2512	-0.2480
495	2001	-0.121	-0.108	-0.1694	-0.1800
496	2002	-0.221	-0.208	-0.1515	-0.3530
Terminal $				82	100
Value		10.4	8.4	43	40
mean %		10.8%	9.6%	18.1%	20.1%
	$\hat{\sigma}$	17%	15%	24%	35%
geomean % (annualized)		9.4%	8.5%	15.6%	15.2%

Number of samples

Cs0
DAC-2 Σ(w) =2: 2.5

Cs25
DAC-2 Σ(w) =2: 1.1

The “if-only” TV, avoiding the 2000-2001 cs25 drawdowns

We have added an arrow to the “what-if” TV amount, prior to 2000 drawdowns. It is more charitable to call it moot, than to observe its practicality in highlighting the amount lost in 3 years. Since it seemed this approach was not leading to large improvements, the DAC-2 Σ(w)=1 approach was implemented, after some time spent finding r* to keep the weights adding to 0.5.

Table 3.5 SP-500 simugram, comparison DAC-2 Sum2, DAC-2 Sum1

				DAC-2 Σ(w)=2		DAC-2 Σ(w)=1	
N	Year	Wil 5000	Geomkt	cs0	cs25	cs0	cs25
397	1977	-0.070	-0.120	0.0499	0.1154	0.0898	0.1020
396	1978	0.040	0.006	0.2187	0.1766	0.1232	0.3390
396	1979	0.193	0.118	0.3175	0.1491	0.2288	0.2123
395	1980	0.522	0.226	0.6366	0.8245	0.7549	0.9427
392	1981	-0.084	-0.091	-0.1646	-0.1776	-0.1906	-0.1171
394	1982	0.129	0.157	0.3895	-0.1186	0.4200	0.3866
395	1983	0.187	0.188	0.2660	0.3464	0.3176	0.2533
396	1984	-0.013	-0.012	-0.0919	-0.2007	-0.0665	-0.2030
395	1985	0.272	0.271	0.3547	0.5319	0.3260	0.6145
387	1986	0.125	0.165	0.2364	0.3719	0.2864	0.4931
396	1987	-0.007	0.012	0.0552	0.1404	0.0391	-0.1499
396	1988	0.133	0.125	0.0050	0.0775	-0.0133	0.0148
388	1989	0.249	0.264	0.3369	0.2193	0.3309	0.3310
386	1990	-0.093	-0.068	-0.0958	-0.1131	-0.1017	-0.0995
383	1991	0.303	0.256	0.4709	0.7787	0.5199	0.6404
382	1992	0.062	0.049	0.1293	-0.1186	0.0912	-0.1039
383	1993	0.086	0.097	0.3771	0.7768	0.4661	1.0043
380	1994	-0.025	-0.007	0.0347	-0.0308	0.0104	-0.0556
496	1995	0.334	0.336	0.3477	0.2715	0.2837	0.3636
494	1996	0.188	0.217	0.2692	0.3236	0.2510	0.5357
494	1997	0.292	0.276	0.2504	0.4180	0.3702	0.6503
498	1998	0.217	0.214	0.3421	0.4638	0.4125	0.6273
494	1999	0.220	0.222	0.5342	0.7760	0.5600	0.8686
498	2000	-0.118	-0.094	-0.2512	-0.2480	-0.2967	-0.3892
495	2001	-0.121	-0.108	-0.1694	-0.1800	-0.1562	-0.2444
496	2002	-0.221	-0.208	-0.1515	-0.3530	-0.1948	-0.4214
Terminal $				82	100	93	310 “if-only” TV
Value		10.4	8.4	43	40	44.4	83
mean %		10.8%	9.6%	18.1%	20.1%	18.7%	25.4%
	$\hat{\sigma}$	17%	15%	24%	35%	27%	41%
geomean % (annualized)		9.4%	8.5%	15.6%	15.2%	15.7%	18.5%

Number of samples

Cs0
DAC-2 Σ(w) =2: 2.5
DAC-2 Σ(w) =1: 3.3

Cs25
DAC-2 Σ(w) =2: 1.1
DAC-2 Σ(w) =1: 1.4

No bottom line progress appears to be made on CS0, using our SP-100 “rule of thumb” for TV deviation, and the deviation of returns appears to increase. Conspicuously missing is any order of magnitude improvement in CS25, which is insufficient remuneration for the tremendous drawdowns we see in the last 3 years. As we shall see in the future research section, it does look like the hedging program would be paying off, whereas it would not have for most of the SP-100 experiments since there was only one year when the portfolio suffered greater than a 20% decline (with CS0).

3.2.2 3-Group Portfolio (DAC-3)

If the N-M were still stalling with N=250, then surely things would improve if we used $k = 3$ for a dimension of 165. This involved selecting 3 groups, but also required finding r* for $3*26 = 78$ groups. Results for CS0 and CS25 are in the table below.

Table 3.6 SP-500 simugram, DAC-2, DAC-3 comparisons. Samples in CS0, DAC-3 Σ(w)=1: 5.7; DAC-2 Σ (w)=2: 2.5; DAC-2 Σ (w)=1: 3.3; samples in CS25, DAC-3 Σ (w)=1: 5.6; DAC-2 Σ (w)=2: 1.1; DAC-2 Σ (w)=1: 1.4.

				DAC-2				**DAC-3**	
				Σ(w)=2		Σ(w)=1		Σ(w)=1	
Time	N	Year	Geomkt	cs0	cs25	cs0	cs25	cs0	cs25
2.82	397	1977	-0.120	0.0499	0.1154	0.0898	0.1020	0.0718	0.0671
2.65	396	1978	0.006	0.2187	0.1766	0.1232	0.3390	0.1793	0.2933
2.65	396	1979	0.118	0.3175	0.1491	0.2288	0.2123	0.2097	0.1236
0.82	395	1980	0.226	0.6366	0.8245	0.7549	0.9427	0.7028	0.6589
1.92	392	1981	-0.091	-0.1646	-0.1776	-0.1906	-0.1171	-0.1712	-0.1142
1.92	394	1982	0.157	0.3895	-0.1186	0.4200	0.3866	0.2810	0.2194
0.77	395	1983	0.188	0.2660	0.3464	0.3176	0.2533	0.3279	0.4677
2.52	396	1984	-0.012	-0.0919	-0.2007	-0.0665	-0.2030	-0.1012	0.0362
1.38	395	1985	0.271	0.3547	0.5319	0.3260	0.6145	0.3792	0.6333
1.9	387	1986	0.165	0.2364	0.3719	0.2864	0.4931	0.2661	0.4772
1.08	396	1987	0.012	0.0552	0.1404	0.0391	-0.1499	0.1027	-0.0071
1.82	396	1988	0.125	0.0050	0.0775	-0.0133	0.0148	-0.0051	0.0189
2.52	388	1989	0.264	0.3369	0.2193	0.3309	0.3310	0.2939	0.3491
1.25	386	1990	-0.068	-0.0958	-0.1131	-0.1017	-0.0995	-0.0985	-0.1196
1.53	383	1991	0.256	0.4709	0.7787	0.5199	0.6404	0.5708	0.7892
0.93	382	1992	0.049	0.1293	-0.1186	0.0912	-0.1039	0.1187	-0.0907
2.65	383	1993	0.097	0.3771	0.7768	0.4661	1.0043	0.4497	0.8956
0.97	380	1994	-0.007	0.0347	-0.0308	0.0104	-0.0556	0.0447	-0.0616
3.63	496	1995	0.336	0.3477	0.2715	0.2837	0.3636	0.2964	0.2492
	494	1996	0.217	0.2692	0.3236	0.2510	0.5357	0.3159	0.3846
2.88	494	1997	0.276	0.2504	0.4180	0.3702	0.6503	0.3863	0.7334
2.96	498	1998	0.214	0.3421	0.4638	0.4125	0.6273	0.4541	0.6164
3	494	1999	0.222	0.5342	0.7760	0.5600	0.8686	0.6011	0.7238
3.32	498	2000	-0.094	-0.2512	-0.2480	-0.2967	-0.3892	-0.2515	-0.2588
	495	2001	-0.108	-0.1694	-0.1800	-0.1562	-0.2444	-0.1297	-0.1140
2.18	496	2002	-0.208	-0.1515	-0.3530	-0.1948	-0.4214	-0.2219	-0.4170
Terminal $			13	82	100	92	310	108	296
Value			8.4	43	40	44	83	55	113
mean %			9.6%	18.1%	20.1%	18.7%	25.4%	19.5%	25.2%
$\hat{\sigma}$			15%	24%	35%	27%	41%	26%	36%
geomean % (annualized)			8.5%	15.6%	15.2%	15.7%	18.5%	16.6%	20.0%

It appears that some progress might be induced. In both CS0 and CS25 the final TV's might be changing in an upward trend. The deviation in CS0 and CS25 are both a little less. More importantly, the recent years' drawdown is considerable reduced, certainly in CS25. And finally, we have obtained what appears to be a significant outperformance of the SP-100 equal-weighted portfolio with TV=39 over the 26 years. This has been our informal benchmark for CS0 throughout the process. Additionally, albeit in CS25, we have also achieved a 20% return, matching the SP-100 CS0 simugram return. There was still along way to go, though, if we were required to use the Wilshire Equal-weighted index as benchmark, especially with its +31% return in 2001, since its 26-year TV is 106.

It must be admitted that by this time we were using the robustness properties of r* heavily. But the time required to complete one year had gone down to 2 hours, which is reflected in the increase number of sample available in the DAC-3 results. Let us review the summary results:

Table 3.7 Summary results, SP-500 vs. SP-100 simugram vs. market, 1977-2002

Portfolio	Description	k	Σ(w)	TV	r_bar	sigma	r_geo
sp500	Market Index Returns	1	1	8.3	9.5%	15%	8.5%
Dow 30	Market Index Returns	1	1	8.2	9.6%	16%	8.4%
Wilshire	Market Index Returns	1	1	10.4	10.8%	17%	9.4%
Geomarket	Market Index Returns	1	1	8.4	9.6%	15%	8.5%
sp100	Simugram mktcap	1	1	11.3	0.111	17.6%	0.098
sp100	Simugram equal-wt	1	1	39.3	0.165	17.9%	0.152
sp100	Mean Baseline, cs0	1	1	245.8	0.269	29.1%	0.236
sp500	DAC-2 sum1, cs0	2	1	44.4	0.187	26.6%	0.157
sp500	DAC-3 sum1, cs0	3	1	54.8	0.195	26.2%	0.166
sp500	DAC-2 sum2, cs0	2	2	43.5	0.181	23.9%	0.156
sp500	DAC-2 sum2, cs25	2	2	40.1	0.201	34.8%	0.152
sp500	DAC-2 sum1, cs25	2	1	82.7	0.254	41.1%	0.185
sp500	DAC-3 sum1, cs25	3	1	113.4	0.252	36.2%	0.200

3.2.3 5-Group Portfolio (DAC-5)

In preparing for the 2-pass optimization, based on the decision to have *N*=100 in each group, it was necessary to split Ω^0 in 5 groups. This was again performed randomly (other schemes are discussed in chapter 4).

Table 3.8 SP-500 simugram, comparison DAC1/DAC-2/DAC-3, DAC-5 Sum5. Average samples in DAC-5 CS0: 12.5, and in CS25: 2.9

			DAC-k, Σ(w)=1, CS0			DAC-5, Σ(w)=5	
N	Year	Geomkt	DAC1	DAC-2	DAC-3	cs0	cs25
397	1977	-0.120	0.090	0.0898	0.0718	-0.0043	0.0614
396	1978	0.006	0.117	0.1232	0.1793	0.1098	0.1226
396	1979	0.118	0.251	0.2288	0.2097	0.2431	0.2175
395	1980	0.226	0.663	0.7549	0.7028	0.4524	0.7030
392	1981	-0.091	-0.186	-0.1906	-0.1712	-0.0656	-0.1951
394	1982	0.157	0.390	0.4200	0.2810	0.3035	0.3720
395	1983	0.188	0.311	0.3176	0.3279	0.2170	0.2782
396	1984	-0.012	-0.085	-0.0665	-0.1012	-0.0353	-0.1173
395	1985	0.271	0.304	0.3260	0.3792	0.2739	0.3478
387	1986	0.165	0.277	0.2864	0.2661	0.1747	0.2705
396	1987	0.012	0.090	0.0391	0.1027	0.0470	0.0451
396	1988	0.125	-0.001	-0.0133	-0.0051	0.0324	0.0675
388	1989	0.264	0.299	0.3309	0.2939	0.1757	0.2963
386	1990	-0.068	-0.137	-0.1017	-0.0985	-0.0503	-0.0733
383	1991	0.256	0.607	0.5199	0.5708	0.3195	0.5184
382	1992	0.049	0.055	0.0912	0.1187	0.1163	0.0345
383	1993	0.097	0.418	0.4661	0.4497	0.2178	0.4088
380	1994	-0.007	-0.009	0.0104	0.0447	-0.0030	-0.0247
496	1995	0.336	0.270	0.2837	0.2964	0.2838	0.2443
494	1996	0.217	0.262	0.2510	0.3159	0.2260	0.2780
494	1997	0.276	0.314	0.3702	0.3863	0.2921	0.2456
498	1998	0.214	0.454	0.4125	0.4541	0.3001	0.4115
494	1999	0.222	0.772	0.5600	0.6011	0.2495	0.7122
498	2000	-0.094	-0.310	-0.2967	-0.2515	-0.1309	-0.2394
495	2001	-0.108	-0.196	-0.1562	-0.1297	-0.0911	-0.1828
496	2002	-0.208	-0.221	-0.1948	-0.2219	-0.1258	-0.2088

Terminal $	13	90	93	108	30	73
Value	8.4	39.0	44	55	21	36
mean %	9.6%	18.5%	18.7%	19.5%	13.6%	17.7%
$\hat{\sigma}$	15%	28%	27%	26%	16%	27%
geomean % (annualized)	8.5%	15.1%	15.7%	16.6%	12.4%	14.8%

Recall that the total return for k groups with equal allocation in each of $\frac{C}{k}$ when $\Sigma w_j = 1$ is $\bar{r}$. Having to average group returns, no matter how stellar one of them may be, is taking its toll at $k = 5$, as seem in the new results. Fortunately, this was not the intent of using 5 groups. The intent was to re-optimize on the combined VSL from each group. Those results follow under the moniker "ReDAC".

Table 3.9 SP-500 simugram, comparison DAC-1/DAC-2/DAC-3/DAC-5 and ReDAC, Σ(w)=1. Samples in DAC-5 CS0: 12.5, and in CS25: 2.9. Samples in ReDAC CS0: 16.5; CS25: 5.7

			DAC-k, Σ(w)=1, CS0			**DAC-5, Σ(w)=5**		**ReDAC-5**	
N	Year	Geomkt	DAC1	DAC-2	DAC-3	cs0	cs25	cs0	cs25
397	1977	-0.120	0.090	0.0898	0.0718	-0.0043	0.0614	0.0748	0.1027
396	1978	0.006	0.117	0.1232	0.1793	0.1098	0.1226	0.1166	0.4626

We should compare these with a summary of the DAC-k Sum k results. DAC-3 sum 3 tests were not calculated, although they were generated, since the objective by that time was to get to ReDAC.

Table 3.10 SP-500 simugram, CS0, comparison DAC1/DAC-2/DAC-5 and ReDAC, $\Sigma(w) = k$

			DAC-k, Σ(w)=k, CS0			**Σ(w)=1**
N	Year	Geomkt	DAC1	DAC-2	DAC-5	ReDAC
397	1977	-0.120	0.090	0.0499	-0.0043	0.0748
396	1978	0.006	0.117	0.2187	0.1098	0.1166
396	1979	0.118	0.251	0.3175	0.2431	0.1860
395	1980	0.226	0.663	0.6366	0.4524	0.6687
392	1981	-0.091	-0.186	-0.1646	-0.0656	-0.1760
394	1982	0.157	0.390	0.3895	0.3035	0.3576
395	1983	0.188	0.311	0.2660	0.2170	0.3112
396	1984	-0.012	-0.085	-0.0919	-0.0353	-0.0775
395	1985	0.271	0.304	0.3547	0.2739	0.2962
387	1986	0.165	0.277	0.2364	0.1747	0.2275
396	1987	0.012	0.090	0.0552	0.0470	0.0824
396	1988	0.125	-0.001	0.0050	0.0324	0.0109
388	1989	0.264	0.299	0.3369	0.1757	0.2929
386	1990	-0.068	-0.137	-0.0958	-0.0503	-0.1019
383	1991	0.256	0.607	0.4709	0.3195	0.5299
382	1992	0.049	0.055	0.1293	0.1163	0.0815
383	1993	0.097	0.418	0.3771	0.2178	0.3992
380	1994	-0.007	-0.009	0.0347	-0.0030	0.0061
496	1995	0.336	0.270	0.3477	0.2838	0.2620
494	1996	0.217	0.262	0.2692	0.2260	0.2409
494	1997	0.276	0.314	0.2504	0.2921	0.3203
498	1998	0.214	0.454	0.3421	0.3001	0.4322
494	1999	0.222	0.772	0.5342	0.2495	0.6254
498	2000	-0.094	-0.310	-0.2512	-0.1309	-0.2806
495	2001	-0.108	-0.196	-0.1694	-0.0911	-0.1322
496	2002	-0.208	-0.221	-0.1515	-0.1258	-0.1879
Terminal $		13	90	82	30	73
Value		8.4	39.0	43	21	37
mean %		9.6%	18.5%	18.1%	13.6%	17.6%
	$\hat{\sigma}$	15%	28%	24%	16%	25%
geomean % (annualized)		8.5%	15.1%	15.6%	12.4%	14.9%

In comparing these last two tables, we see that when $\Sigma w = 1$, the TV increases as k increases. When $\Sigma w = k$ it appears the TV is not significantly changed until k=5, when it drops precipitously. Unfortunately we do have data for DAC-5 Sum 1, neither for DAC-3 Sum 3, both of which would be required to make definitive statements on the direction of these results (see chapter 4).

3.3 Performance Issues

Our goal is really to show that the simugram portfolio weight selection technique applied to a large portfolio of stocks can outperform the returns of the index itself. This we have done, both for the SP-100 and the SP-500. Except for DAC-5, the SP-500 simugram returns provide a 400% improvement in TV, and an annualized return that is 175-200% that of the market index. These are reviewed below; each group sorted by volatility.

Table 3.11 Summary results, SP-500/100 Simugram vs. market and themselves, 1977-2002.

Portfolio	Description	k	Σ(w)	TV	r_bar	sigma	r_geo
sp100	SP Equal-wt	1	1	39.3	0.165	17.9%	0.152
sp100	SP Mktcap-wt	1	1	11.3	0.111	17.6%	0.098
Wilshire	Market Index Returns	1	1	10.4	10.8%	17.4%	9.4%
Dow 30	Market Index Returns	1	1	8.2	9.6%	15.8%	8.4%
Geomarket	Market Index Returns	1	1	8.4	9.6%	15.1%	8.5%
sp500	Market Index Returns	1	1	8.3	9.5%	14.8%	8.5%
sp100	Mean Baseline, cs0	1	1	246	0.269	29.1%	0.236
sp500	DAC-3 sum1, cs0	3	1	54.8	0.195	26.2%	0.166
sp500	DAC-2 sum1, cs0	2	1	44.4	0.187	26.6%	0.157
sp500	Baseline, 1-group	1	1	39.0	0.185	28.2%	0.151
sp500	DAC-2 sum2, cs0	2	2	43.5	0.181	23.9%	0.156
sp500	ReDAC, cs0	5	1	36.9	0.176	25.2%	0.149
sp500	DAC-5 sum5, cs0	5	5	21.0	0.136	16.2%	0.124
sp100	Mean Baseline, cs25	1	1	5,125	0.387	53.0%	0.295
sp500	DAC-2 sum1, cs25	2	1	82.7	0.254	41.1%	0.185
sp500	ReDAC, cs25	5	1	50.6	0.229	39.8%	0.163
sp500	DAC-3 sum1, cs25	3	1	113.4	0.252	36.2%	0.200
sp500	DAC-2 sum2, cs25	2	2	40.1	0.201	34.8%	0.152
sp500	DAC-5 sum5, cs25	5	5	35.9	0.177	26.5%	0.148

However, when comparing the relative results for the SP-100 with those of the SP-500 experiments, a severe underperformance issue has arisen. Since the SP-100 portfolio is completely contained in that for the SP-500, this should not be. The "outperformance" of the SP-500 against the SP-100 is highlighted in figure 3.3

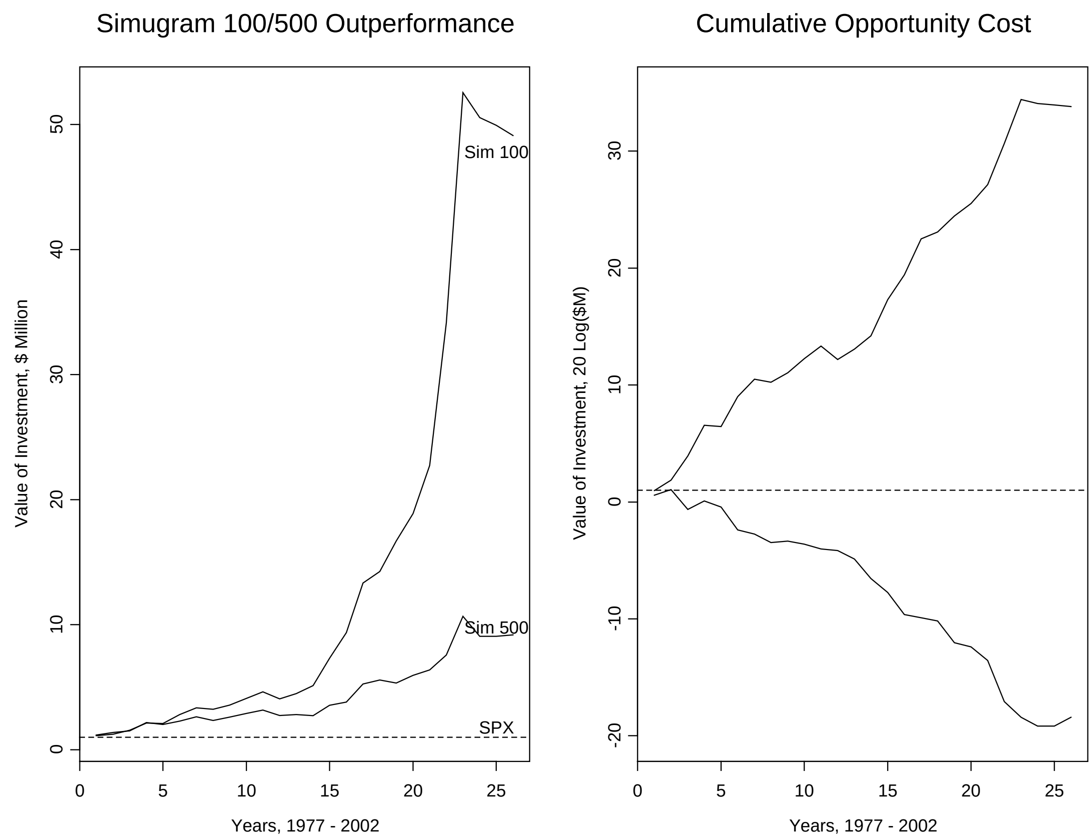

Figure 3.3 Simugram Outperformance of SP-500 SPX market index, and Opportunity cost

The left panel is self-explanatory, plotting $M in terminal value. The right panel though shows some opportunity costs of not using the SP-100 simugram trading system. If one were investing in a derivative fund earning the difference between the SP-100 CS0 and the SP-500 DAC-3 systems, which is possible and explained in many sources such as [3, 54], then one's investment would have lost 88%, shown on the bottom portion of panel B. Moreover, one would have lost the gain to the upside. So we must do better.

Laboring under the assumption that there is nothing unique in a superior sense about the collection of stocks known as the SP-100, the disappointing results for the 2-pass optimization leave us with the following options:

a. Continue the approach with a DAC-4 or DAC-5, and $\Sigma w = 1$. Since TV and return seem monotonic in *k*, this might work, especially if the cause is Nelder-Mead stalling at an unfortunately small dimension of *N*=102. This would involve multiple trials to ensure that the *r** value is acceptable in each. The r^* would get progressively smaller. If one broke up a 100-stock portfolio into 100 sub portfolios of 1 stock each, with a required maximum allocation of 100%, then even if a stock returned a whopping 20% in the year, its weighted (Σw) return is 0.012, meaning that unless one's "minimum acceptable return" in the tail is 0.012, then the sum of weights constraint would break. However, we would not be performing this to hyperbole, but just to k=4 or 5. This is a viable alternative and should be pursued to completion.

b. Use the DAC-k with $\Sigma w = k$, and introduce discretionary judgment on which of the k portfolios will do the best in the coming year, and weight the allocation toward that one. The problem with the $\Sigma w = k$ system is that one has to take the average of all k portfolios returns; clearly, if one could know which of the k portfolios would have the highest return, then that would be the one to pick. This violates the premise of the system, that human intervention is excluded. There are many proprietary alternative funds available, and many offer similar returns in the 20-30% range, and with even higher volatility than our baseline systems.

c. As is said in the world of linear programming, "find another optimizer."[1] There may be some merit to this option. The fact that there is improvement as *k* increases suggests that larger blocks do indicate stalling is occurring. Until the results of the DAC-4/5 have been examined, we cannot say if the improvement is due to the optimizer or to some virtue of diversification. We would like to see commensurate results as the SP-100 (say $150-200) with the DAC-4 or 5 using Nelder-Mead. Assuming this can be done, then if there were a better optimizer, and it could achieve as good results on a group 500 stocks; the overhead with doing this with the SP-1000 or the Russell-2000, or the Wilshire itself, would be considerably reduced.

[1] See for example the frequently asked question (FAQ) list at http://www.uni-giessen.de/faq/archiv/linear-programming-faq/msg00000.html, or any other practical linear programming text.