Lecture 16 Outline:

1. From Supervised to Unsupervised Analysis.

2. Principal Components Analysis.
   - Interpretations & Uses.
     - Data Visualization.
     - Pattern Recognition.
     - Dimension Reduction.
   - Models & Optimization Problems.
     - Matrix Factorization: Frobenius norm loss.
     - Covariance: Eigendecomposition of empirical covariances.
   - Solution via the SVD.
     - Properties of the SVD.
     - SVD & PCs / PC Loadings or Directions.
   - Identifiability & Uniqueness.
   - Center the data first? Scaling?
   - Amount of Variance Explained.
     - Proportion of variance explained.
     - How to choose number of components?

3. Nuclear Norm Penalty.
   - Solution via singular value thresholding.

4. Real Example: NCI microarray Data.

5. PCA in high-dimensions.
   - RMT: PCA inconsistent when $p \gg n$.

6. Sparse PCA.
   - Optimization problems.
     - Semi-definite programming approach.
     - Alternating penalized least squares approach.
   - Solutions.
   - Deflation approach.
   - Properties.
   - Proportion of variance explained.
   - Real data example: NCI microarray data.

7. Kernel PCA.
   - Simulated examples.