Laplace's Probability of Sunrise

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Let X be the number of days in which the sun has risen out of N days, and Y be a binary variable indication whether the sun rises tomorrow or not (i.e. $Y = 1$ indicates sunrise).

Let θ be the probability of sunrise on any given day, and let's assume that before observing X we had no information about what values of θ are likely. That is, assume that the prior distribution for θ is $Unif(0, 1)$.

We're interested in $P(Y = 1 | X = N) = \int_0^1 P(Y = 1 | \theta) P(\theta | X = N) \, d\theta$
$= \int_0^1 \theta \frac{P(\theta | X=N)}{\int_0^1 P(\theta | X=N) d\theta} d\theta = \int_0^1 \theta \frac{\theta^N}{\int_0^1 \theta^N d\theta} d\theta = \int_0^1 \theta^{N+1} (N+1) d\theta = \frac{N+1}{N+2}$

Note that, as absurd as the result may seem, it is relying on the assumption that we have no "extra" knowledge about sunrises. That is, if we didn't happen to know that gravitational laws dictate that the sun will rise for sure, this might not be such an unreasonable result.