CIMPA-UCR

Análisis Factorial de Correspondencias Simples

(AFC)

CIMPA-UCR

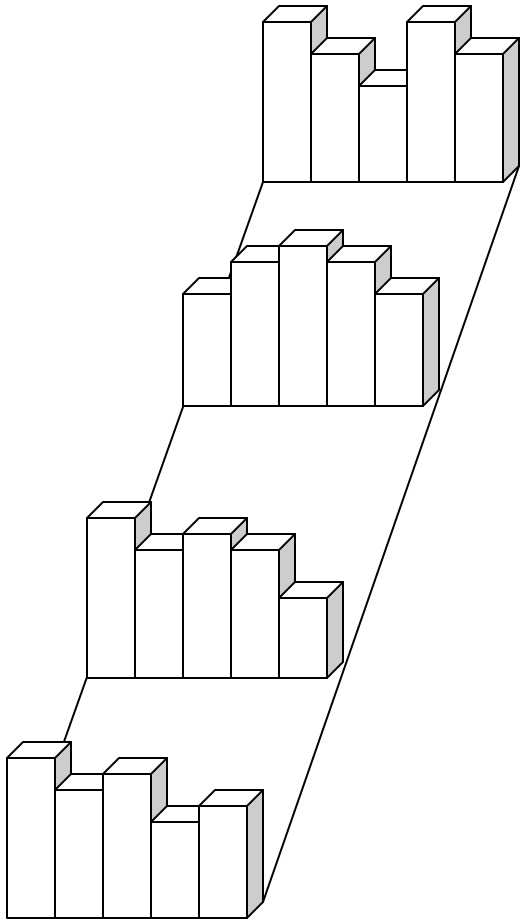
TABLAS DE CONTINGENCIA

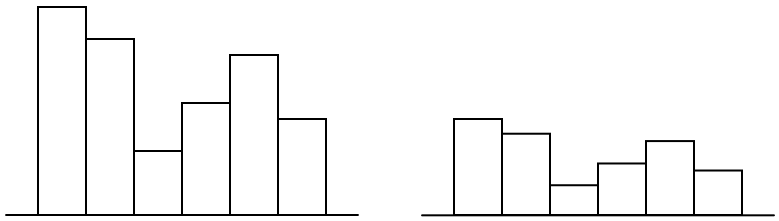
Tipo de empleo

Tipo de salario	Funcionario	Agricultor	Obrero/ Empleado	Otros	
Bajo	11	14	107	75	207
Medio	1	10	60	31	102
Alto	23	2	16	40	81
	35	26	183	146	

CIMPA-UCR

Histograma de las filas





Mismo comportamiento respecto a las columnas

CIMPA-UCR

ANÁLISIS DE CORRESPONDENCIAS

J.P. Benzécri, 1962
Pearson, Guttman, Fisher, Burt, Hayashi, Gifi

Análisis de una tabla de contingencia:
$\boldsymbol{x}$: cualitativa, con modalidades $x^1, x^2, ..., x^p$
$\boldsymbol{y}$: cualitativa, con modalidades $y^1, y^2, ..., y^q$

	y^1	...	y^k	...	y^q	
x^1	n_{11}	...	n_{1k}	...	n_{1q}	$n_{1\bullet}$
	⋮		⋮		⋮	
x^j	n_{j1}	...	n_{jk}	...	n_{jq}	$n_{j\bullet}$
	⋮		⋮		⋮	
x^p	n_{p1}	...	n_{pk}	...	n_{pq}	$n_{p\bullet}$
	$n_{\bullet 1}$		$n_{\bullet k}$		$n_{\bullet q}$	n

Efectivo de modalidad j de $\boldsymbol{x}$:

$$n_{j\bullet} = \sum_{k=1}^{q} n_{jk}$$

Efectivo de modalidad k de $\boldsymbol{y}$:

$$n_{\bullet k} = \sum_{j=1}^{p} n_{jk}$$

Población total:

$$n = \sum_{j=1}^{p} \sum_{k=1}^{q} n_{jk}$$

PERFILES: FILA Y COLUMNA

Atenuar disparidades en los tamaños de las modalidades

Perfil – fila $\boldsymbol{j}$*:*

$$\boxed{n_{j1} \ldots n_{jk} \ldots n_{jq}} \quad n_{j\bullet} \quad \Rightarrow \quad \boxed{\frac{n_{j1}}{n_{j\bullet}} \ldots \frac{n_{jk}}{n_{j\bullet}} \ldots \frac{n_{jq}}{n_{j\bullet}}} \quad 1$$

Ej: $\boxed{\frac{11}{207} \quad \frac{14}{207} \quad \frac{107}{207} \quad \frac{75}{207}} = \boxed{0.053 \quad 0.068 \quad 0.517 \quad 0.362}$

Perfil marginal fila: $\boxed{\frac{n_{\bullet 1}}{n} \ldots \frac{n_{\bullet k}}{n} \ldots \frac{n_{\bullet q}}{n}} = \boxed{f_{\bullet 1} \cdots f_{\bullet k} \cdots f_{\bullet q}}$

CIMPA-UCR

PERFILES: FILA Y COLUMNA

Atenuar disparidades en los tamaños de las modalidades

Perfil – columna ***k****:*

$$\begin{bmatrix} n_{1k} \\ \vdots \\ n_{jk} \\ \vdots \\ n_{pk} \end{bmatrix} \Rightarrow \begin{bmatrix} n_{1k}/n_{\bullet k} \\ \vdots \\ n_{jk}/n_{\bullet k} \\ \vdots \\ n_{pk}/n_{\bullet k} \end{bmatrix} = \begin{bmatrix} f_{1k}/f_{\bullet k} \\ \vdots \\ f_{jk}/f_{\bullet k} \\ \vdots \\ f_{pk}/f_{\bullet k} \end{bmatrix}$$

$n_{\bullet k}$ $\qquad$ 1

Ej: $\begin{bmatrix} 11/35 \\ 1/35 \\ 23/35 \end{bmatrix} = \begin{bmatrix} 0.314 \\ 0.029 \\ 0.657 \end{bmatrix}$

Perfil marginal columna: $\begin{bmatrix} n_{1\bullet}/n \\ \vdots \\ n_{j\bullet}/n \\ \vdots \\ n_{p\bullet}/n \end{bmatrix} = \begin{bmatrix} f_{1\bullet} \\ f_{j\bullet} \\ f_{p\bullet} \end{bmatrix}$

NUBE DE PERFILES - FILA

$\mathcal{N}_x$:

- p puntos de $\Re^q$: perfiles - fila

$$\left(\frac{n_{j1}}{n_{j\bullet}}, \ldots, \frac{n_{jk}}{n_{j\bullet}}, \ldots, \frac{n_{jq}}{n_{j\bullet}} \right) \in \Re^q$$

- peso del perfil - fila j: $\dfrac{n_{j\bullet}}{n} = f_{j\bullet}$

- métrica: $M_x = diag\left(\dfrac{n}{n_{\bullet k}} \right) = \begin{pmatrix} n/n_{\bullet 1} & & 0 \\ & \ddots & \\ & & n/n_{\bullet q} \end{pmatrix}$

NUBE DE PERFILES - FILA

- Centro de gravedad: $g_x = \begin{pmatrix} f_{\bullet 1} \\ \vdots \\ f_{\bullet q} \end{pmatrix}$

- Distancia de χ^2: $d^2_{\chi^2}(p-f\ j, p-f\ h) = \sum_{k=1}^{q} \frac{n}{n_{\bullet k}} \left(\frac{n_{jk}}{n_{j\bullet}} - \frac{n_{hk}}{n_{h\bullet}} \right)^2$

$$= \sum_{k=1}^{q} \frac{1}{f_{\bullet k}} \left(\frac{f_{jk}}{f_{j\bullet}} - \frac{f_{hk}}{f_{h\bullet}} \right)^2$$

- Inercia: $\mathrm{I}(\mathcal{N}_x) = \sum_{j=1}^{p} \frac{n_{j\bullet}}{n} d^2_{\chi^2}(p-f\ j, g_x)$

Propiedad: $\mathrm{I}(\mathcal{N}_x) = \frac{\chi^2}{n} = \Phi^2$

NUBE DE PERFILES - COLUMNA

$\mathcal{N}_y$:

- q puntos de $\Re^p$: perfiles - columna

$$\left(\frac{n_{1k}}{n_{\bullet k}}, \ldots, \frac{n_{jk}}{n_{\bullet k}}, \ldots, \frac{n_{pk}}{n_{\bullet k}} \right) \in \Re^p$$

- peso del perfil - columna k: $\dfrac{n_{\bullet k}}{n} = f_{\bullet k}$

- métrica: $M_y = diag\left(\dfrac{n}{n_{j\bullet}} \right) =$

$$\begin{pmatrix} n/n_{1\bullet} & & 0 \\ & \ddots & \\ & & n/n_{j\bullet} \\ & \ddots & \\ 0 & & n/n_{p\bullet} \end{pmatrix}$$

NUBE DE PERFILES - COLUMNA

- Centro de gravedad: $g_y = \begin{pmatrix} f_{1\bullet} \\ \vdots \\ f_{p\bullet} \end{pmatrix}$

- Distancia de χ^2: $d^2_{\chi^2}(p-c\ k, p-c\ l) = \sum_{j=1}^{p} \frac{n}{n_{j\bullet}} \left(\frac{n_{jk}}{n_{\bullet k}} - \frac{n_{jl}}{n_{\bullet l}} \right)^2$

$$= \sum_{j=1}^{p} \frac{1}{f_{\bullet k}} \left(\frac{f_{jk}}{f_{\bullet k}} - \frac{f_{jl}}{f_{\bullet l}} \right)^2$$

- Inercia: $\mathrm{I}(\mathcal{N}_y) = \sum_{k=1}^{q} \frac{n_{\bullet k}}{n} d^2_{\chi^2}(p-c\ j, g_y)$

Propiedad: $\mathrm{I}(\mathcal{N}_y) = \frac{\chi^2}{n} = \Phi^2$

ACP DE UNA NUBE DE PERFILES

- Centrar los perfiles *(fila):* $\left(\cdots, \frac{f_{jk}}{f_{j\bullet}} - f_{\bullet k}, \cdots \right)$

$$(columna): \left(\cdots, \frac{f_{jk}}{f_{\bullet k}} - f_{j\bullet}, \cdots \right)$$

- Calcular "varianzas-covarianzas":

$$\mathbf{V}_x = \begin{pmatrix} \cdots & \frac{f_{jk}}{f_{j\bullet}} - f_{\bullet k} & \cdots \\ & \vdots & \end{pmatrix}^t \begin{pmatrix} n_{\bullet 1}/n & & \\ & \ddots & \\ & & n_{\bullet q}/n \end{pmatrix} \begin{pmatrix} \cdots & \frac{f_{jk}}{f_{j\bullet}} - f_{\bullet k} & \cdots \\ & \vdots & \end{pmatrix}$$

$$= \begin{pmatrix} \cdots & \sum_j \left(\frac{n_{jk}\, n_{jk'}}{n\, n_{j\bullet}} \right) - \frac{n_{\bullet k}\, n_{\bullet k'}}{n^2} & \cdots \\ & \vdots & \end{pmatrix}_{q \times q} \qquad \mathbf{V}_y = \ldots_{p \times p}$$

ACP DE UNA NUBE DE PERFILES

- Diagonalizar $R_x = M_x^{1/2} V_x M_x^{1/2}$

$$R_y = M_y^{1/2} V_y M_y^{1/2}$$

- Propiedad Si $H = (h_{jk})$ con

$$= \frac{\qquad}{\sqrt{\qquad \bullet \quad \bullet}} - \frac{\sqrt{\qquad \bullet \quad \bullet}}{\qquad}$$

Entonces: $R_x = H^t H$ y $R_y = H H^t$

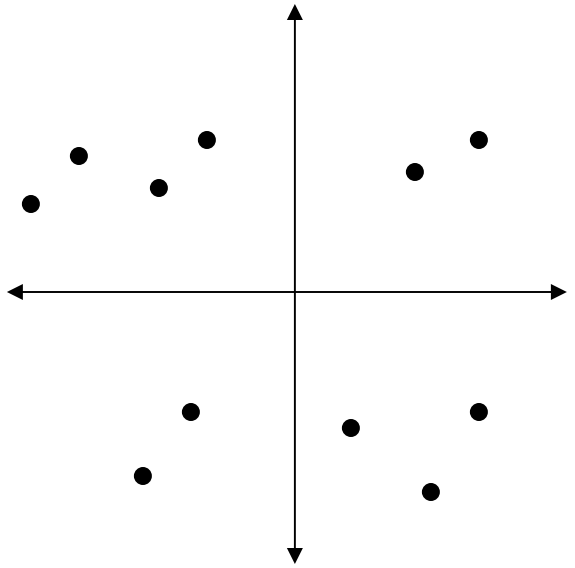
RESULTADOS del AFC

Componentes principales $\mathcal{N}_x$

Componentes principales $\mathcal{N}_y$

⇨ Representación simultánea

Calidad global: $\dfrac{\lambda_1 + \lambda_i + \cdots}{I(\mathrm{N}_x)}$



Calidad de representación puntos: cos^2

Ejes

- % Inercia
- Interpretabilidad

- filas
- columnas

AFC: CALIDAD DE LA REPRESENTACIÓN

- **Contribuciones relativas:** $CTR_\alpha(j) = \frac{1}{\lambda_\alpha} f_{j\bullet} C_\alpha^2(j)$

  Con: λ_α: valor propio λ

  $f_{j\bullet}$: peso del perfil j

  $C_\alpha(j)$: coordenada del perfil j en componente principal α

- **Contribuciones absolutas:** $CTA_\alpha(j) = f_{j\bullet}\, C_\alpha^2(j)$

- **Cosenos cuadrados:** Si $CTR_\alpha(j)$ es grande: perfil j jugó un papel preponderante en el posicionamiento del eje α

FÓRMULAS DE TRANSICIÓN

Propiedad 1: $\boldsymbol{R}_x$, $\boldsymbol{R}_y$ tienen los mismos valores propios

Propiedad 2: si $R_x v_x = \lambda v_x , v_x^t v_x = 1$ y $R_y v_y = \lambda v_y , v_y^t v_y = 1$

$$\Rightarrow \quad v_x = \pm \frac{1}{\sqrt{\lambda}} H^t v_y \qquad v_y = \pm \frac{1}{\sqrt{\lambda}} H \; v_x$$

sean C^1, C^2 Componentes principales del ACP de $\mathcal{N}_x$

D^1, D^2 Componentes principales del ACP de $\mathcal{N}_y$

entonces: $$c_j = \frac{1}{\lambda} \sum_{k=}^{q} \frac{n_{jk}}{n_{j\bullet}} d_k$$

$$d_k = \frac{1}{\lambda} \sum_{j=}^{p} \frac{n_{jk}}{n_{\bullet k}} c_j$$

PROPIEDAD DE EQUIVALENCIA DISTRIBUCIONAL

Si dos perfiles-fila son iguales, entonces se pueden "fundir" y las distancias entre perfiles-fila y entre perfiles-columna no se alteran

Hipótesis: $\forall k = 1,\ldots,q : \dfrac{n_{j'k}}{n_{j'\bullet}} = \dfrac{n_{j''k}}{n_{j''\bullet}}$

Nuevo perfil: $\dfrac{n_{j*k}}{n_{j*\bullet}} = \dfrac{n_{j'k} + n_{j''k}}{n_{j'\bullet} + n_{j''\bullet}}$ con peso $n_{j*\bullet} = n_{j'\bullet} + n_{j''\bullet}$

$$\frac{n_{j*k}}{n_{j*\bullet}} = \frac{n_{j'k} + \dfrac{n_{j'k}\, n_{j''\bullet}}{n_{j'\bullet}}}{n_{j'\bullet} + n_{j''\bullet}} = \frac{n_{j'k}}{n_{j'\bullet}} \left(\frac{n_{j'\bullet} + n_{j''\bullet}}{n_{j'\bullet} + n_{j''\bullet}} \right) = \frac{n_{j'k}}{n_{j'\bullet}}$$

distancias entre perfiles-fila no se alteran

CIMPA-UCR

DISTANCIAS ENTRE PERFILES-COLUMNAS

$$d^2_{\chi^2}(k,k') = \underbrace{\sum_{j=1}^{p} \frac{n}{n_{j\bullet}}\left(\frac{n_{jk}}{n_{\bullet k}} - \frac{n_{jk'}}{n_{\bullet k'}}\right)^2}_{antes} = \underbrace{\sum_{\substack{j=1 \\ j\neq j',j''}}^{p} \frac{n}{n_{j\bullet}}\left(\frac{n_{jk}}{n_{\bullet k}} - \frac{n_{jk'}}{n_{\bullet k'}}\right)^2 + \frac{n}{n_{j^*\bullet}}\left(\frac{n_{j^*k}}{n_{\bullet k}} - \frac{n_{j^*k'}}{n_{\bullet k'}}\right)^2}_{después}$$

Lo que interesa analizar es:

$$\frac{n}{n_{j'\bullet}}\left(\frac{n_{j'k}}{n_{\bullet k}} - \frac{n_{jk'}}{n_{\bullet k'}}\right)^2 + \frac{n}{n_{j''\bullet}}\left(\frac{n_{j''k}}{n_{\bullet k}} - \frac{n_{j''k'}}{n_{\bullet k'}}\right)^2 = \frac{n}{n_{j^*\bullet}}\left(\frac{n_{j^*k}}{n_{\bullet k}} - \frac{n_{j^*k'}}{n_{\bullet k'}}\right)^2$$

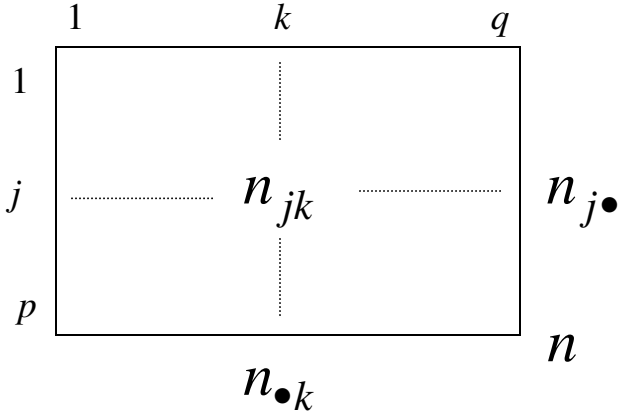
CIMPA-UCR

AFC

Tabla de contingencia

$$D_p = diag(f_{j\bullet}) = M_y^{-1}$$

$$D_q = diag(f_{\bullet k}) = M_x^{-1}$$



Frecuencia:

$$f_{jk} = \frac{n_{jk}}{n}$$

$\downarrow$

F

Nubes de perfiles-fila $\mathcal{N}_x$

Nubes de perfiles-columna $\mathcal{N}_y$

$D_p^{-1}F = \left(\frac{f_{jk}}{f_{j\bullet}}\right)$	Tabla	$D_q^{-1}F^t = \left(\frac{f_{jk}}{f_{\bullet k}}\right)$
$D_q^{-1} = diag\left(\frac{1}{f_{\bullet k}}\right) = M_x$	Métrica χ^2	$D_p^{-1} = diag\left(\frac{1}{f_{j\bullet}}\right) = M_y$
$D_p = diag\left(f_{j\bullet}\right)$	Pesos	$D_q = diag\left(f_{\bullet k}\right)$

CIMPA-UCR

AFC

ACP de $\mathcal{N}_x$***:*** hallar u con $\|u\|=1 \quad tq \quad \underset{u}{Max}\{I^1_{\Delta u}(N_x)\}$

$$I_{\Box u^{\perp}}(N_x) \Box u^t \tilde{D}_q^{\,1} F^t \tilde{D}_p^{\,1} D_p \tilde{D}_p^{\,1} F \tilde{D}_q^{\,1} u \Box u^t M_x V_x M_x u$$

Solución: diagonalizar

$$V_x M_x = F^t D_p^{-1} D_p D_p^{-1} F D_q^{-1} = F^t D L_p^{-1} F D_q^{-1} = S$$

ACP de $\mathcal{N}_y$: diagonalizar

$$V_y M_y = F D_q^{-1} F^t D_p^{-1} = T$$

Comp. Principales:

$$\psi_\alpha = D_p^{-1} F D_q^{-1} u_\alpha$$

$$\varphi_\alpha = D_q^{-1} F^t D_p^{-1} v_\alpha$$

CIMPA-UCR

AFC TABLA DE CONTINGENCIA (1)

- Individuos: 121 pacientes
- Filas: 4 Tipos de drogas (medicamentos)
- Columnas: calificación de los pacientes

	Malo	Regular	Bueno	Muy Bueno	Excelente
A	5	1	10	8	6
B	5	3	3	8	12
C	10	6	12	3	0
D	7	12	8	1	1

	Valores propios	% inercia	% acumulado
λ_1	0.30	78.32	78.32
λ_2	0.08	19.88	98.20
λ_3	0.01	1.80	100.00

AFC TABLA DE CONTINGENCIA (2)

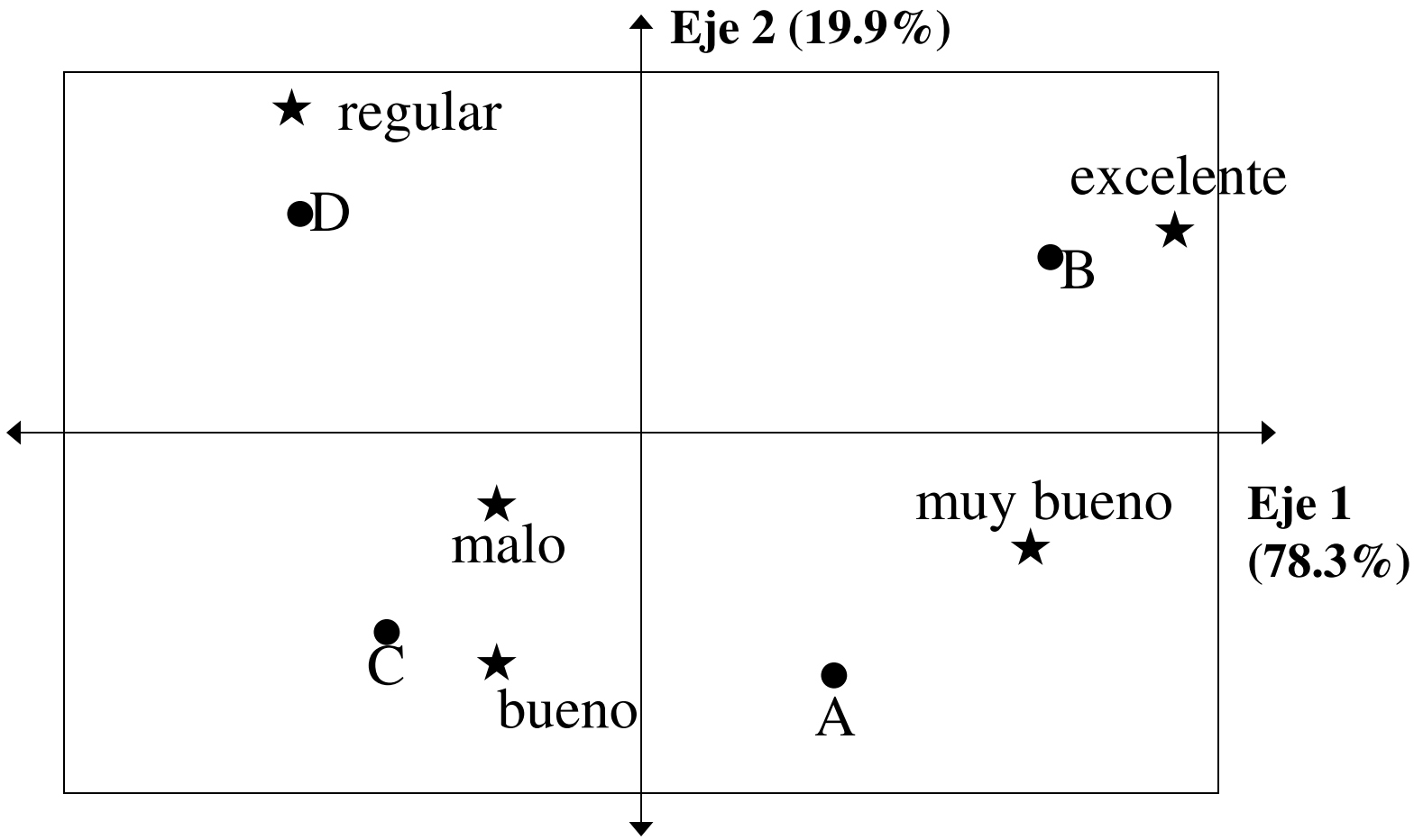
Tabla de componentes principales y cosenos cuadrados

	Eje 1	Eje 2	$Cos^2 1$	$Cos^2 2$	Calidad en el plano
malo	-0.25	-0.07	69.10	6.32	75.42
regular	-0.64	0.44	67.35	32.10	99.45
bueno	-0.27	-0.30	43.68	54.37	98.06
muy bueno	0.58	-0.16	92.29	6.67	98.96
excelente	*0.95*	*0.28*	*92.10*	*7.85*	99.95
A	0.35	-0.30	54.73	40.68	95.41
B	0.70	0.25	88.57	10.81	99.38
C	-0.45	-0.25	74.31	22.09	96.40
D	-0.63	0.31	79.33	19.82	99.14

CIMPA-UCR

AFC TABLA DE CONTINGENCIA (3)

Primer plano principal



AFC DE UNA TABLA DE NOTAS (1)

Tabla de datos: notas escolares de 10 estudiantes en 5 materias, con notas entre 0 y 10

- Componentes positivas
- Filas homogéneas y columnas homogéneas
- Tiene sentido calcular el perfil

Valores Propios

$\lambda_1 = 0.01$ (61.90%)
$\lambda_2 = 0.0053$ (32.62%)
$\lambda_3 = 0.0008$ (4.96%)
$\lambda_4 = 0.00008$ (0.52%)
$\lambda_5 = 0$ (0.00%)

CIMPA-UCR

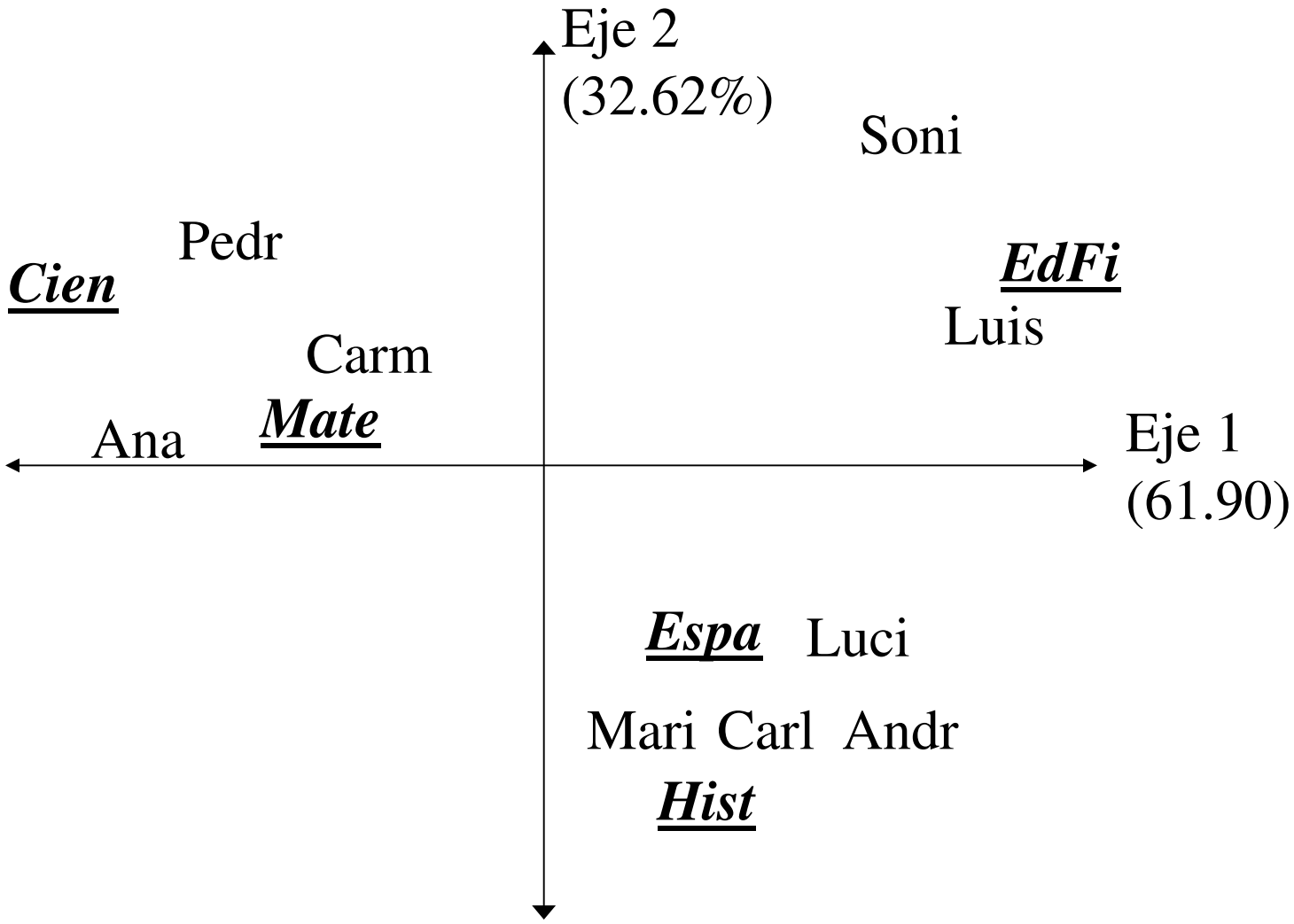
AFC DE UNA TABLA DE NOTAS (2)

Proyecciones y cosenos cuadrados

	Eje 1	Eje 2	$Cos^2 1$	$Cos^2 2$	Calidad
Mate	-0.08	0.01	76.62	0.34	76.96
Cien	-0.15	0.05	86.98	10.59	97.57
Espa	0.04	-0.05	28.12	53.23	81.36
Hist	0.04	-0.10	11.70	79.10	90.80
Edif	0.14	0.10	64.88	34.98	99.85
Luci	0.08	-0.05	57.41	22.83	80.24
Pedr	-0.11	0.06	77.73	21.90	99.64
Carm	-0.07	0.02	88.26	11.21	99.47
Luis	0.13	0.10	54.73	32.94	87.66
Andr	0.09	-0.08	55.16	40.28	95.43
Ana	-0.12	0.00	99.78	0.01	99.79
Carl	0.07	-0.08	39.69	59.25	98.94
Jose	0.15	-0.01	98.97	0.52	99.49
Soni	0.11	0.15	32.10	61.15	93.25
Mari	0.02	-0.08	7.01	91.12	98.13

CIMPA-UCR

AFC DE UNA TABLA DE NOTAS (3)

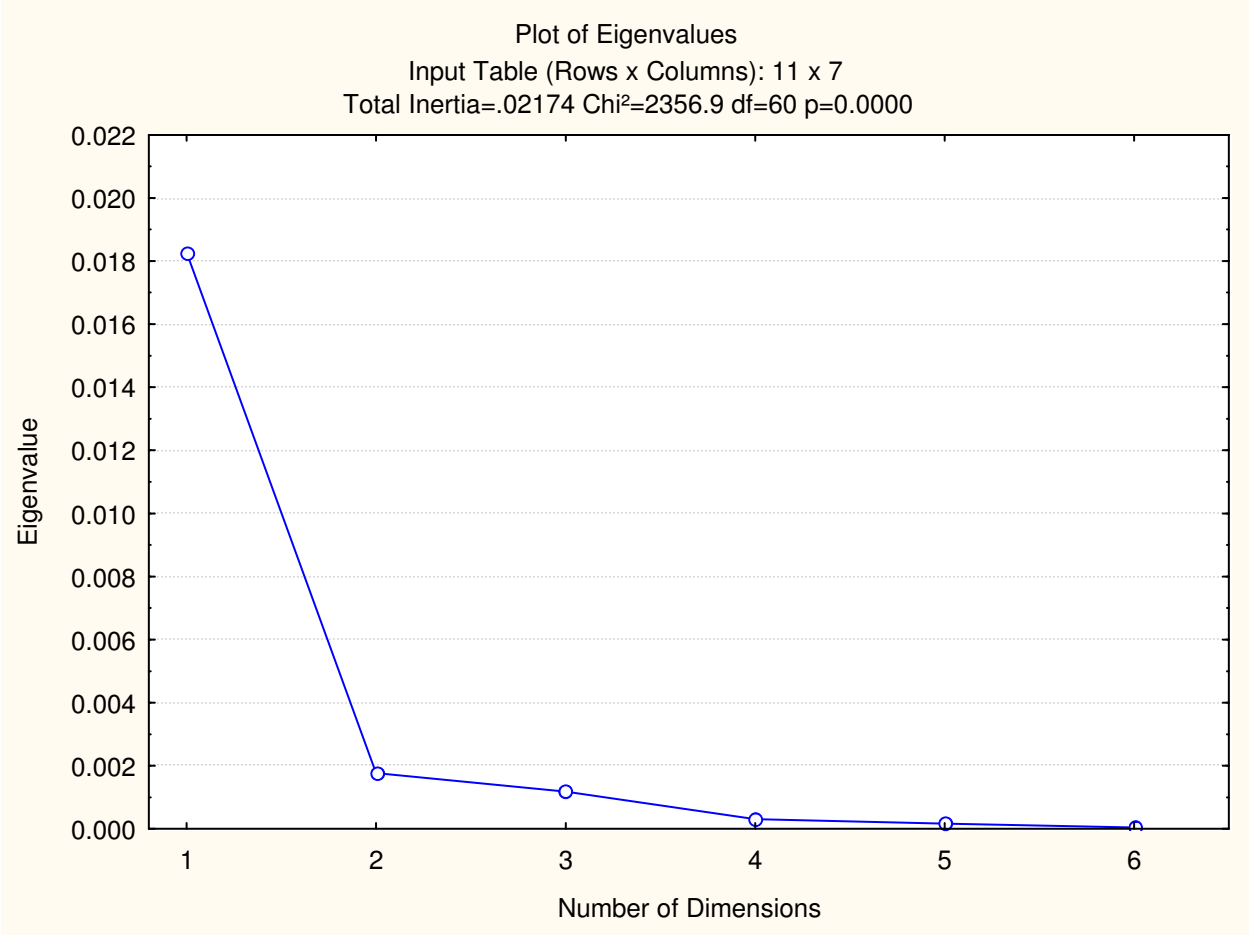


Ejemplo: categorías de los profesores de Mate. en el MEP

	MT1	MT2	MT3	MT4	MT5	MT6	Asp
1993	268	305	751	2500	711	112	1478
1995	348	496	876	2531	643	119	1503
1996	386	660	992	2563	787	162	1564
1998	436	756	1158	2889	901	194	1372
1999	498	859	1317	3524	1118	234	1559
2000	530	857	1303	3857	1188	273	1365
2001	631	1040	1387	4176	1302	330	1629
2002	559	1214	1664	4907	1713	443	1580
2003	621	1227	1809	5531	1927	516	1524
2004	710	1344	1706	6052	2160	520	1587
2005	828	1263	1490	5100	1968	689	1307

CIMPA-UCR

Valores propios



Coordenadas de las filas

	Row	Coor(1)	Coor(2)	Mass	Quality	Rel. Iner	Iner(1)	Cos^2 (1)	Iner(2)	Cos^2 (2)
1993	1	-0.260436	0.122395	0.056505	0.987660	0.217880	0.209771	0.808985	0.478005	0.178675
1995	2	-0.242683	0.009235	0.060112	0.987390	0.165139	0.193776	0.985962	0.002895	0.001428
1996	3	-0.196058	-0.046302	0.065629	0.964026	0.127061	0.138077	0.913099	0.079453	0.050927
1998	4	-0.092428	-0.062281	0.071091	0.962836	0.042180	0.033241	0.662174	0.155718	0.300662
1999	5	-0.070279	-0.033006	0.084034	0.925463	0.025175	0.022718	0.758225	0.051697	0.167239
2000	6	-0.000333	-0.006778	0.086469	0.027869	0.006571	0.000001	0.000067	0.002243	0.027802
2001	7	-0.014162	-0.033340	0.096820	0.629683	0.009279	0.001063	0.096246	0.060774	0.533438
2002	8	0.063756	-0.003096	0.111442	0.703618	0.029678	0.024794	0.701964	0.000603	0.001655
2003	9	0.103734	0.022970	0.121359	0.884678	0.071218	0.071478	0.843328	0.036159	0.041350
2004	10	0.119575	0.041313	0.129884	0.944699	0.101200	0.101646	0.843958	0.125182	0.100742
2005	11	0.178499	-0.010506	0.116655	0.838298	0.204618	0.203436	0.835404	0.007271	0.002894

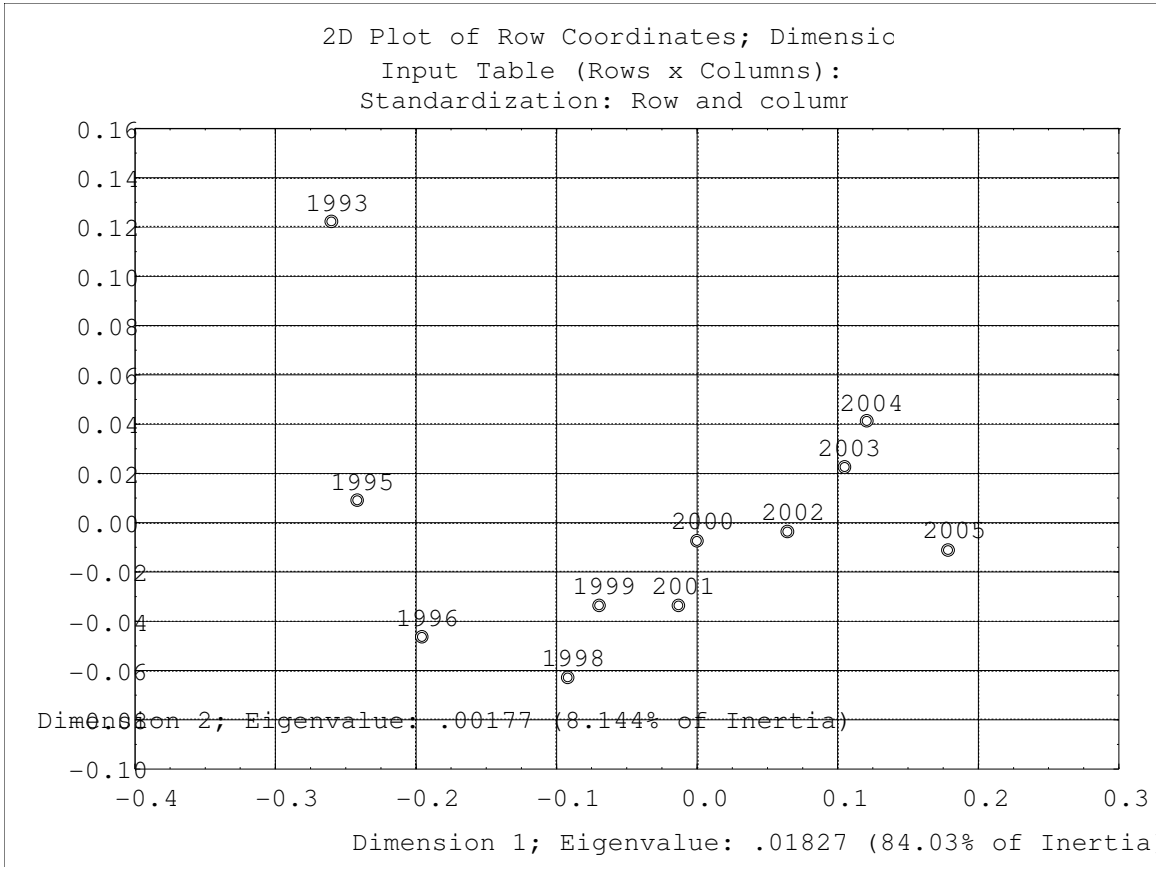
CIMPA-UCR

Coordenadas de las columnas

	Column	Coord(1)	Coord(2)	Mass	Quality	Relative	Iner(1)	$Cos^2(1)$	Iner(2)	$Cos^2(2)$
MT1	1	0.023429	-0.069109	0.053645	0.384890	0.034133	0.001612	0.039677	0.144684	0.345213
MT2	2	0.085916	-0.089841	0.092447	0.938752	0.069987	0.037351	0.448425	0.421364	0.490327
MT3	3	-0.030011	-0.041697	0.133334	0.496506	0.032596	0.006573	0.169429	0.130910	0.327077
MT4	4	0.030131	0.031251	0.402502	0.866388	0.040264	0.020001	0.417387	0.221980	0.449001
MT5	5	0.124424	0.032246	0.133011	0.965264	0.104700	0.112707	0.904511	0.078103	0.060753
MT6	6	0.284201	-0.004148	0.033137	0.848174	0.145159	0.146496	0.847993	0.000322	0.000181
Asp	7	-0.284968	0.005544	0.151923	0.990312	0.573160	0.675262	0.989937	0.002637	0.000375

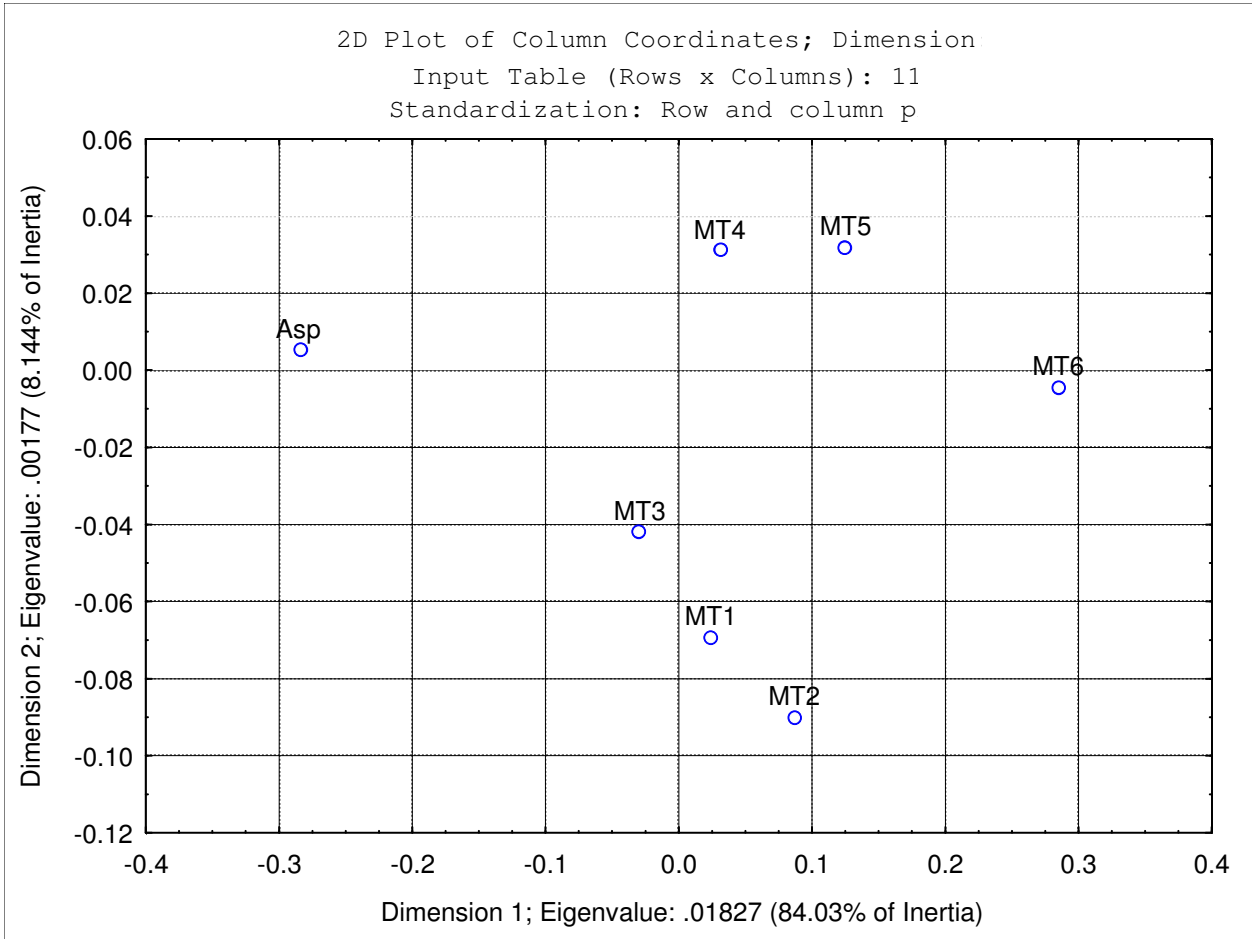
CIMPA-UCR

Años

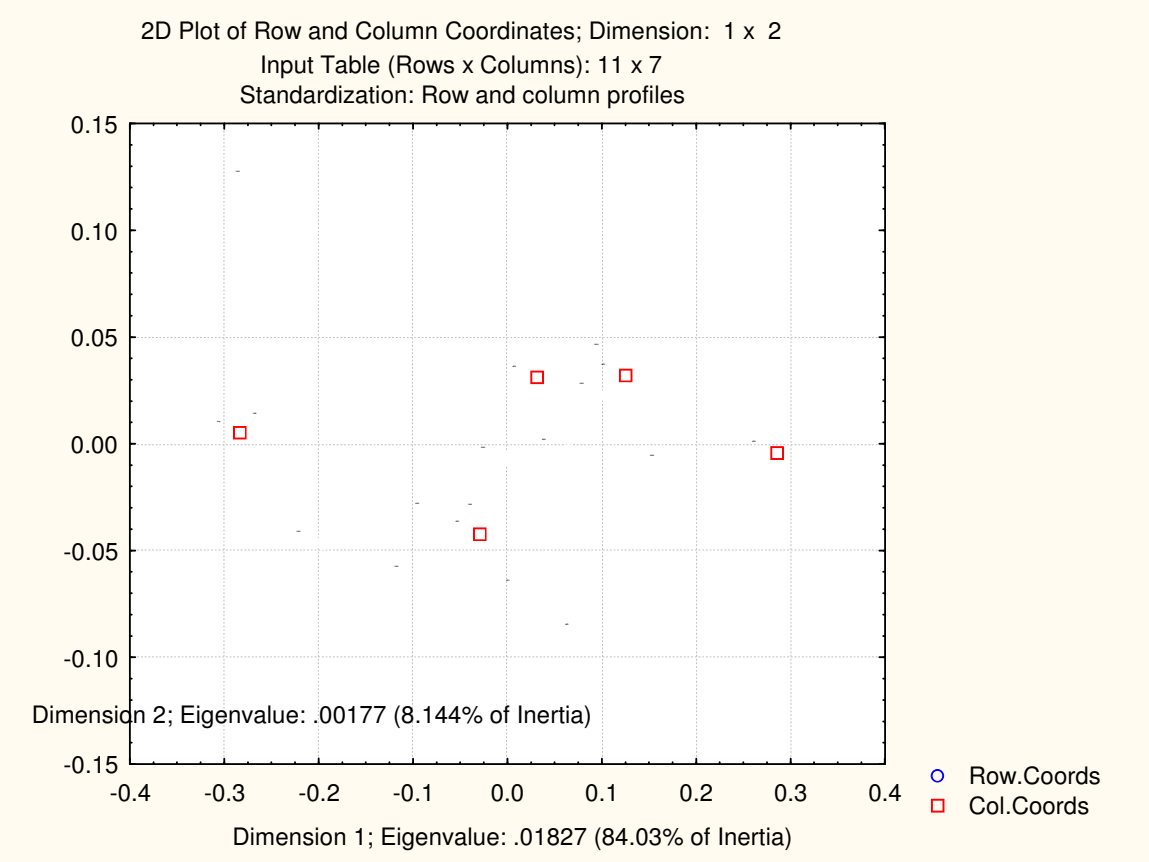


CIMPA-UCR

Categorías



Plano principal



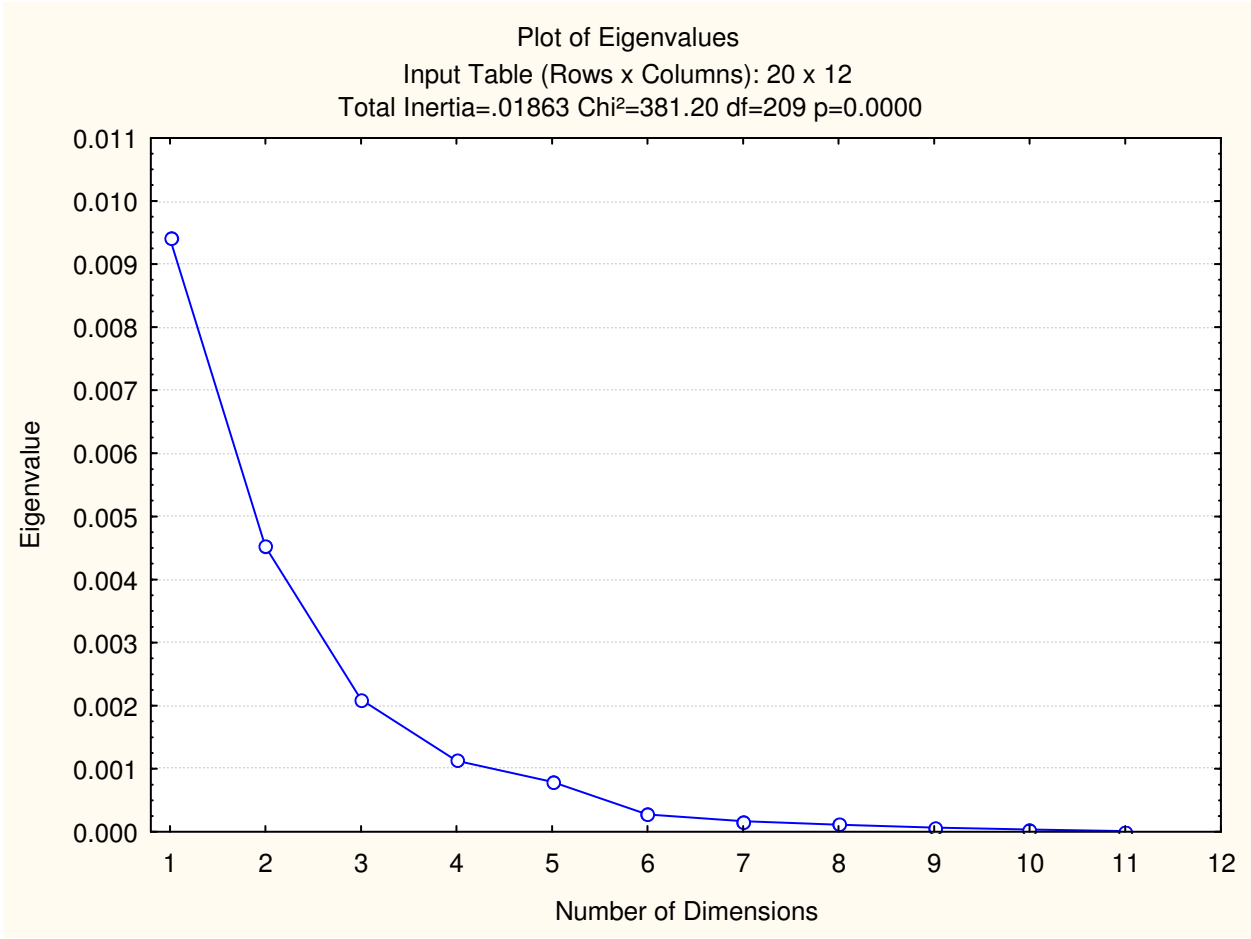
CIMPA-UCR

MEP: Número de profesores por materia y según región

	Espa-	EstSoc	Mate	Cien	Qui	Bio-	Fsi-	Ingles	Frances	EdFis	Musica	Otro
San José	452	384	385	323	177	185	138	339	146	193	148	1043
Desamparados	88	105	98	93	32	30	32	87	45	73	41	481
Puriscal	31	37	34	34	15	16	13	35	15	23	11	143
Prez Zeledón	89	101	98	77	43	41	39	92	39	70	32	399
Alajuela	144	164	155	129	57	59	61	135	59	108	57	565
San Ramón	78	96	84	73	25	27	25	85	35	51	22	275
San Carlos	80	85	87	58	48	48	41	92	30	71	31	397
Upala	32	31	31	23	20	21	18	36	11	17	11	97
Cartago	131	159	143	126	48	50	46	136	61	100	45	582
Turrialba	36	39	38	29	14	15	14	33	13	20	11	140
Heredia	160	189	188	141	56	56	51	171	76	108	138	743
Liberia	48	52	47	44	20	18	15	45	18	34	21	190
Nicoya	34	34	36	25	17	16	14	41	12	24	11	122
Santa Cruz	38	41	40	29	19	19	14	59	15	30	16	167
Cañas	28	32	33	30	13	13	12	27	13	19	11	105
Puntarenas	68	76	77	63	32	31	27	81	32	55	26	307
Coto	75	83	87	66	33	36	32	79	25	34	19	308
Aguirre	28	34	30	27	14	14	11	36	10	13	8	140
Limón	97	102	107	92	39	42	42	103	41	52	30	361
Guápiles	64	73	68	63	22	23	22	81	20	38	20	249

CIMPA-UCR

Valores propios



Coordenadas de las filas

	Row	Coordin.	Coordin.	Mass	Quality	Relative	Inertia	Cosine	Inertia	Cosine
San Jos	1	-0.165555	0.042639	0.191204	0.988724	0.303426	0.557329	0.927220	0.076873	0.061504
Desamparados	2	0.157108	-0.005255	0.058881	0.868868	0.089899	0.154561	0.867897	0.000360	0.000971
Puriscal	3	0.032777	-0.049877	0.019888	0.577991	0.006580	0.002272	0.174325	0.010941	0.403665
Prez Zeledn	4	0.044677	-0.047592	0.054728	0.717266	0.017454	0.011617	0.335993	0.027412	0.381273
Alajuela	5	0.016157	-0.004948	0.082727	0.097158	0.013052	0.002297	0.088827	0.000448	0.008330
San Ramn	6	-0.013624	-0.020387	0.042805	0.047689	0.028971	0.000845	0.014722	0.003934	0.032967
San Carlos	7	0.056378	-0.082525	0.052187	0.402915	0.069457	0.017640	0.128209	0.078596	0.274706
Upala	8	-0.167095	-0.060454	0.017005	0.599469	0.048084	0.050492	0.530084	0.013743	0.069385
Cartago	9	0.067446	-0.028995	0.079502	0.654415	0.035151	0.038460	0.552332	0.014781	0.102083
Turrialba	10	0.008386	-0.036620	0.019643	0.353930	0.004205	0.000147	0.017638	0.005825	0.336292
Heredia	11	0.109643	0.165489	0.101490	0.960343	0.223583	0.129752	0.292955	0.614659	0.667388
Liberia	12	0.037326	0.019658	0.026973	0.433721	0.005942	0.003996	0.339540	0.002305	0.094181
Nicoya	13	-0.041521	-0.053947	0.018861	0.398862	0.011765	0.003458	0.148383	0.012139	0.250479
Santa Cruz	14	0.032332	-0.045394	0.023797	0.144257	0.027506	0.002646	0.048552	0.010844	0.095705
Caas	15	-0.035681	-0.001109	0.016418	0.204539	0.005492	0.002223	0.204342	0.000004	0.000197
Puntarenas	16	0.046925	-0.038721	0.042756	0.715302	0.011877	0.010012	0.425543	0.014177	0.289759
Coto	17	-0.010164	-0.075439	0.042854	0.419434	0.031782	0.000471	0.007478	0.053933	0.411956
Aguirre	18	0.056446	-0.090131	0.017835	0.449801	0.024075	0.006043	0.126716	0.032041	0.323085
Limn	19	-0.029987	-0.035487	0.054141	0.388633	0.016144	0.005178	0.161903	0.015077	0.226730
Gupiles	20	0.012048	-0.038510	0.036306	0.124180	0.025555	0.000560	0.011071	0.011907	0.113109

Coordenadas de las columnas

	Column	Coordin.	Coordin.	Mass	Quality	Relative	Inertia	Cosine	Inertia	Cosine
Espa-	1	-0.151309	0.034535	0.088004	0.926578	0.122816	0.214268	0.880698	0.023211	0.045880
EstSoc	2	-0.029310	0.010416	0.093672	0.226369	0.021495	0.008558	0.200988	0.002247	0.025380
Mate-	3	-0.048522	0.016313	0.091180	0.628222	0.020419	0.022830	0.564427	0.005366	0.063795
Cien-	4	-0.048793	0.008297	0.075495	0.211369	0.046970	0.019114	0.205429	0.001149	0.005940
Qu-	5	-0.154972	-0.052712	0.036355	0.686619	0.076164	0.092853	0.615418	0.022339	0.071201
Bio-	6	-0.177524	-0.054083	0.037137	0.790294	0.086881	0.124464	0.723175	0.024021	0.067119
Fsi-	7	-0.097696	-0.075270	0.032592	0.624109	0.042642	0.033083	0.391637	0.040835	0.232472
Ingles	8	-0.028561	-0.035665	0.087613	0.176688	0.055576	0.007600	0.069035	0.024645	0.107653
Frances	9	-0.013872	0.048399	0.034987	0.245023	0.019432	0.000716	0.018601	0.018124	0.226422
EdFis	10	0.061859	-0.026742	0.055363	0.239845	0.056281	0.022530	0.202080	0.008755	0.037765
Musica	11	0.055129	0.320583	0.034645	0.933755	0.210762	0.011198	0.026820	0.787382	0.906935
Otro	12	0.111824	-0.023862	0.332959	0.971470	0.240562	0.442785	0.929160	0.041926	0.042310

CIMPA-UCR

Regiones

Materias

CIMPA

Plano principal