Statistical methods for financial models driven by Lévy processes

Part IV: Some Non-parametric Methods for Lévy Models

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Outline

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Set-up

1. The price process S_t of an asset is assumed to follow a Geometric Lévy model:

$$S_t = S_0 e^{X_t}, \quad \text{where } X_t \text{ is a Lévy Process.}$$

2. Data consists of discrete-time observations of the process:

$$S_{t_0}, S_{t_1}, \dots, S_{t_n} \iff X_{t_0}, X_{t_1}, \dots, X_{t_n}$$

3. We assume a non-parametric model for X. Concretely, we assume that the "Lévy measure" ν of X has a density $p(x)$:

$$\nu(dx) = p(x)dx.$$

4. Goal: To use the data to estimate (or predict) the function p.

Tools

1. The function p can be approximately recovered from integrals of the form
$$\beta(\varphi) := \int \varphi(x)p(x)dx,$$
by taking test functions φ on certain classes; e.g. indicator functions, splines, wavelet basis, etc.

2. p controls the jump behavior of the process:
$$\mathbb{E}\, \tfrac{1}{T} \sum_{s\leq T} \mathbf{1}_{[a,b]}(\Delta X_s) = \int \mathbf{1}_{[a,b]}(x)\, p(x)dx$$
$$\Downarrow$$
$$\mathbb{E}\, \tfrac{1}{T} \sum_{s\leq T} \varphi\,(\Delta X_s) = \int \varphi(x)p(x)dx$$

3. p determines also the "short-term behavior" of X:
$$\frac{1}{t}\mathbb{E}\left[\varphi\left(X_t\right)\right] \xrightarrow{t\downarrow 0} \int \varphi(x)p(x)dx.$$

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Idea of the Estimation Method

1. Consider
$$\hat{\beta}_T(\varphi) := \frac{1}{T}\sum_{s\leq T}\varphi(\Delta X_s)$$
2. The SLLN implies that
$$\hat{\beta}_T(\varphi) \stackrel{T\to\infty}{\longrightarrow} \beta(\varphi) := \int \varphi(x)p(x)dx.$$
3. Drawback: Neither the size of the jumps ΔX_t nor the times are observable from discrete observations $X_{t_0}, \ldots, X_{t_n}$ of the process.

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The estimation method

Woerner (2003) and FL (2004)

Estimators:

$$\hat{\beta}_n(\varphi) := \frac{1}{t_n} \sum_{i=1}^{n} \varphi(X_{t_i} - X_{t_{i-1}});$$

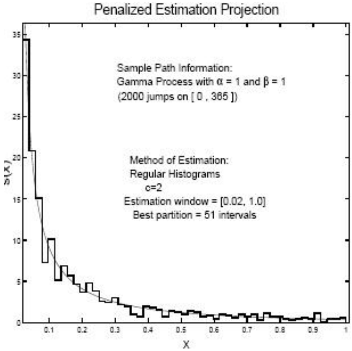
Properties: [Woerner 03, FL & Houdré 06; FL 07]

If φ satisfies some regularity conditions $(\star)$, then as $t_n \to \infty$ and $\delta_n := \max\{t_i - t_{i-1}\} \to 0$,

1. $\mathbb{E}\left\{\hat{\beta}_n(\varphi) - \beta(\varphi)\right\}^2 \longrightarrow 0$
2. $\hat{\beta}_n(\varphi) \xrightarrow{P} \beta(\varphi)$
3. $\sqrt{T_n}\left(\hat{\beta}_n(\varphi) - \beta(\varphi)\right) \xrightarrow{\mathfrak{D}} \mathcal{N}(0, \beta(\varphi^2))$, provided that $\delta_n \sqrt{t_n} \to 0$.

The Gamma Lévy process

1. Model: Pure-jump Lévy process with Lévy density $p(x) = \frac{\alpha}{x} e^{-x/\beta} \mathbf{1}_{\{x>0\}}$.
2. Histogram like estimators: Outside the origin.



Performance

- Least-square fit:

  Fit the model $\frac{\alpha}{x} e^{-x/\beta}$ (using least-squares) to the histogram estimator: $\hat{\alpha}_{LSE} = 0.93$ and $\hat{\beta}_{LSE} = 1.055$ (vs. $\hat{\alpha}_{MLE} = 1.01$ and $\hat{\beta}_{MLE} = 0.94$)

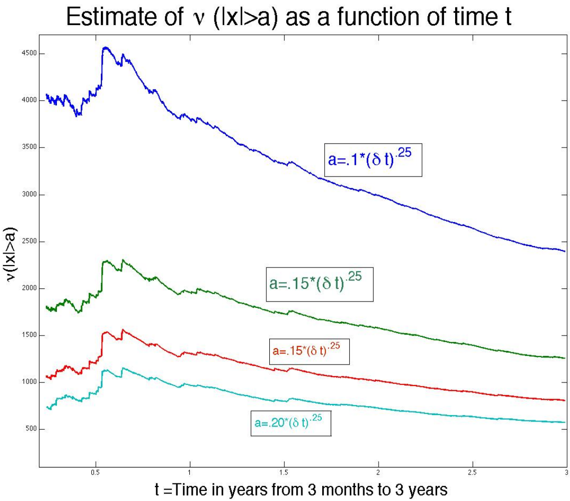
- Sampling distribution

  Means and standard errors of $\hat{\alpha}_{LSE}$ and $\hat{\beta}_{LSE}$ based on 1000 repetitions

Δt	PPE-LSF		MLE	
.1	0.81 (0.06)	1.40 (0.50)	1.001 (0.01)	0.99 (0.05)
.01	0.92 (0.08)	1.12 (0.31)	1.007 (0.07)	0.99 (0.08)
.001	0.93 (0.08)	1.13 (0.34)	1.007 (0.07)	0.99 (0.08)

Recovery of the Lévy measure $\nu(|x| \geq a)$ in time

Estimation based on 5-sec. sampling observations of INTEL from Jan. 2003 to Dec. 2005



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1. The model:

$$S_t = S_0 e^{X_t}, \quad X_t = bt + W_{\tau(t)} + J_{\tau(t)},$$

$\{W_t\}_{t\geq 0}$ is a Wiener process

$\{J_t\}_{t\geq 0}$ is an indep. pure-jump Lévy process with Lévy measure ν.

$$\tau(t) = \int_0^t r_s ds, \quad \text{(Random Clock)}$$

$$dr_t = \alpha(m - r_t)dt + \gamma\sqrt{r_t}\, dW'_t, \quad \text{(Speed of the random clock)}$$

2. Observations: Sampling of the log return process $X_t = \log(S_t/S_0)$:

- $X_0, X_{\delta}, \dots, X_{n\delta}$ with $n\delta \leq T_n$ (Regular sampling);

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   - $X_0, X_\delta, \ldots, X_{n\delta}$ with $n\delta \leq T_n$ (Regular sampling);
   - $X_{t_0^n}, \ldots, X_{t_n^n}$ with $0 = t_0^n < \cdots < t_n^n = T_n$ (Irregular sampling);

Statistical Problems

1. Non-parametric estimation based on high-frequency data:
   - Recovery of the random clock $\tau(t) = \int_0^t r(u)du$ (e.g. Winkel (2001), Woerner (2007), FL. (2009), Aït-Sahalia & Jacod (2009)).
   - Recovery of the "jump-acivity index".
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Recovery of the clock based on high-frequency data

1. Winkel (2001): X infinite jump-activity Lévy and $\tau(t)$ independent continuous increasing:

$$\hat{\tau}_\alpha(T) := \frac{1}{\int_{|x|\geq\alpha} s(x)dx} \cdot \#\{t \leq T : \underbrace{|\Delta X_t|}_{\text{Jump}} \geq \alpha\},$$

2. Figueroa (2009) & Aït-Sahalia & Jacod (2009):

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Recovery of the jump activity index β

1. Suppose that $\int_{|x|\geq\alpha} s(x)dx \sim c_0\alpha^{-\beta}$, as $\alpha \to 0$, for some $\beta \in (0,2)$.
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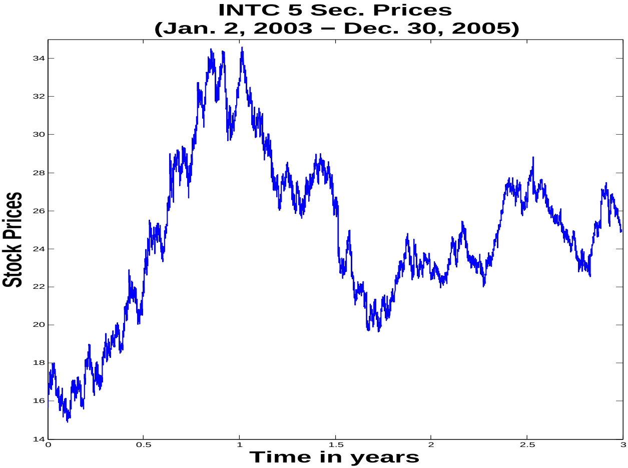

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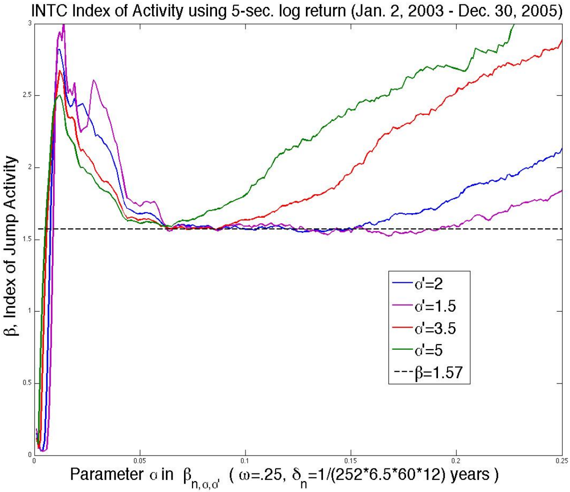

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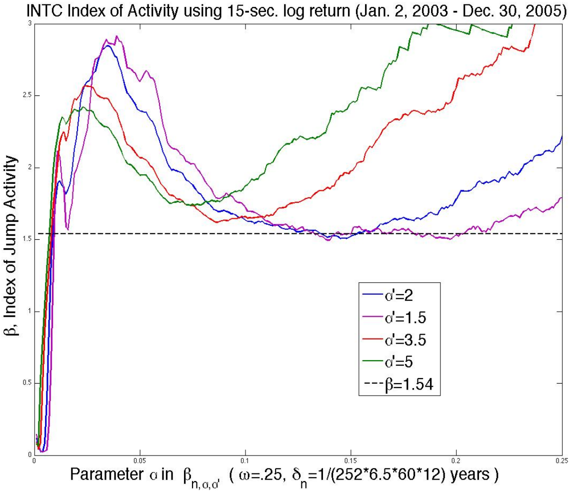
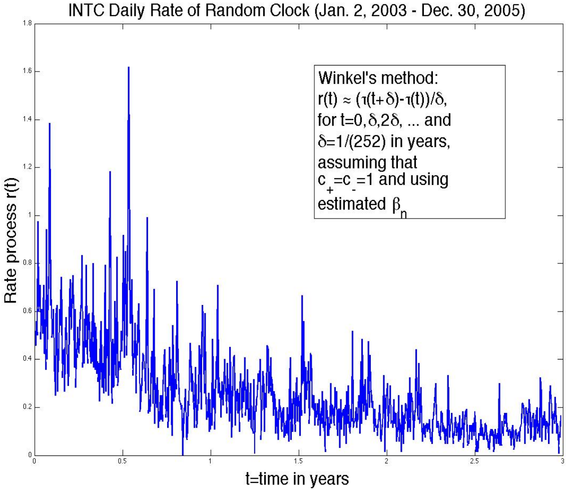


Recovery of the intensity $\{v_t\}_{t\geq 0}$ of random clock



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Conclusions:

In the context of a Time-changed Lévy model,

1. we develop a consistent estimation scheme for the Lévy measure ν, the random clock τ, and the index of activity β using high-frequency data.
2. We obtain central limit theorems for the estimators.

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2. We obtain central limit theorems for the estimators.

Important open problems:

1. Data-driven calibration of the estimator parameters for small samples?
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For Further Reading I

Figueroa-López.
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Advances in Applied Probability, 2009.

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