Applications of concentration of measure in signal processing

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April 30, 2010

Contents

1 Notation

Notation

- X - random variable or vector
- $\mathbb{E}[X]$ - expected value of random variable X.
- $\mathrm{Var}(X)$ - Variance of random variable X.
- $\Pr[\mathbb{A}]$ - Probability of the event $\mathbb{A}$.
- $\|\cdot\|_2$, $\|\cdot\|_1$ - Euclidean norm, Absolute sum norm.

2 Introduction

Abstract: This talk presents applications of concentration of measure phenomena in the emerging field of Compressive Sensing (CS). CS builds on the premise that a signal having a sparse representation in some basis can be recovered from a small number of linear measurements of that signal. Many of the most effective constructions for the linear measurement operator involve random matrices, and at the heart of much analysis in CS is a precise statistical characterization of the product of a random matrix with a sparse signal. Building on a simple concentration of measure inequality, for example, it is possible to generalize the Johnson-Lindenstrauss lemma and ensure an approximate distance-preserving embedding for an entire family of sparse signals. This "Restricted Isometry Property" for the measurement operator has been shown

in CS to permit stable recovery of sparse signals from small numbers of measurements. We will also discuss a related problems in Compressive Signal Processing (CSP), in which the goal is not to recover (high-complexity question) a sparse signal (low-dimensional model) but rather to answer low-complexity questions about arbitrary high-dimensional signals. We will discuss how concentration of measure inequalities can be used to give performance guarantees for problems such as compressive detection and estimation.

Much thanks to Justin Romberg for permission to use the indicated figures in this presentation.

2.1 References

This is a gentle introduction to Compressive Sensing that covers some of the same material as in this talk.

E. Candès and M. Wakin, "People hearing without listening: An introduction to compressive sampling," *IEEE Signal Processing Magazine* vol. 25, no. 2, pp. 21-30, 2008.

The following are some of the first papers to articulate the process of Compressive Sensing. The first paper gives a bound on the number of random measurements necessary for recovery using ℓ_1 optimization that is derived by arguing about the probability of existence of a feasible dual vector. The second paper shows that Compressive Sensing is nearly optimal - no other linear mapping followed by any decoding method could yield lower reconstruction error (up to a constant factor) over classes of compressible signals.

E. Candès, J. Romberg, and T. Tau, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on information theory*, vol. 52, no. 2, pp. 489-509, 2006.

D. Donoho, "Compressed sensing," *IEEE Transactions on Information Theory*, vol. 52, no. 4, pp. 12891306, 2006.

This wonderfully succinct paper gives bounds on recovery error using basis pursuit (ℓ_1 regularization) in both the noise-free and nosy case, under the RIP assumption.

E. Candès, "The restricted isometry property and its implications for compressed sensing," *Comptes rendus-Mathématique*, vol. 346, no. 9-10, pp. 589-592, 2008.

Although this talk will focus on ℓ_1 recovery, other methods of recovery also exist. Some other methods of recovery are discussed in these papers.

J. A. Tropp, "Greed is good: Algorithmic results for sparse approximation," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2231–2242, 2004.

E. J. Candès, M. B. Wakin and S. P. Boyd, "Enhancing sparsity by reweighted ℓ_1 minimization," *Journal of Fourier Analysis and Applications*, vol. 14, no. 5, pp. 877-905, 2008.

Rick Chartrand, "Exact reconstructions of sparse signals via nonconvex minimization," *IEEE Signal Proc. Lett., vol. 14 no.10 pp. 707-710, 2007.*

Hossein Mohimani, Massoud Babaie-Zadeh, Christian Jutten, "Fast Sparse Representation based on Smoothed L0 norm," in: *proceedings of 7th International Conference on Independent Component Analysis and Signal Separation* (ICA2007), LNCS 4666, September 2007, London, pp. 389-396.

This paper gives a proof that random measurement matrices can satisfy the RIP property with $O(S\log(N/S))$ using concentration of measure arguments.

R. Baraniuk, M. Davenport, R. DeVore, and M. Wakin, "A simple proof of the restricted isometry property for random matrices," *Constructive Approximation*, vol. 28, no. 3, pp. 253-263, 2008.

The previous paper uses “Johnson-Lindenstrauss Embedding” to create measurement matrices. The following is some of the relevant literature concerning the Johnson-Lindenstrauss lemma.

P. Frankl and H. Maehara, “The Johnson-Lindenstrauss lemma at the sphericity of some graphs,” *Journal of Combinatorial Theory, Ser. B,* vol. 44, no. 3, pp.355-362, 1988

P. Indyk and R. Motwani, “Approximate nearest neighbors: towards removing the curse of dimensionality”, *30th Annual ACM Symposium on Theory of Computing*, Dallas, TX, pp. 604-613, 1998.

S. Dasgupta and A. Gupta, “An elementary proof of the Johnson-Lindenstrauss lemma”, Technical Report 99-006, UC Berkeley, March 1999.

D. Achlioptas, “Database-friendly random projections: Johnson- Lindenstrauss with binary coins, Journal of Computer and System Sciences, vol. 66, no. 4, pp. 671687, 2003.

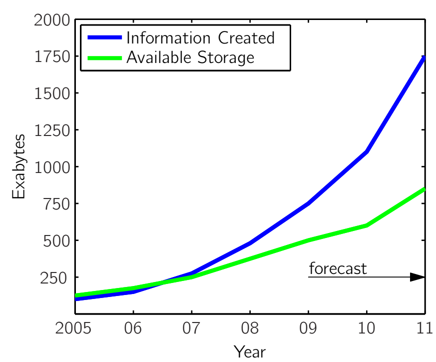
Compressive Sensing is closely related to basis pursuit and lasso.

R. Tibshirani, “Regression shrinkage and selection via the lasso,” *Journal of the Royal Statistical Society, Series B*, vol. 58, no. 1, pp. 267-288, 1996.

S.S. Chen, D. L. Donoho and M. A. Saunders, “Atomic decomposition by basis pursuit,” *SIAM review,* vol. 43, no. 1, pp. 129-159, 2001.

2.2 Motivation

Signal Processing in the Age of the Data Flood

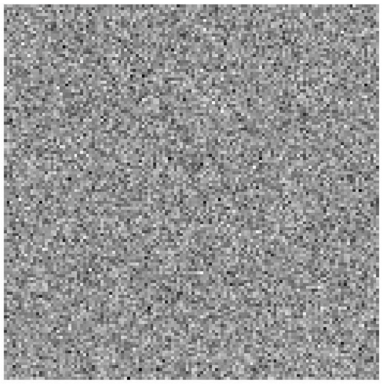


Reference: Economist magazine, Feb 25, 2010.

- Exabyte = 2^{60} bits.
- We have passed the point where all data created can be stored
- LHC generates 40 Tb every second.
- Other bottlenecks
  - acquisition
  - transmission
  - analysis

Not all length-N signals are created equal

- What is the class of "typical images"?



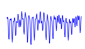

- "Typical" signals contain degrees of freedom S less than N

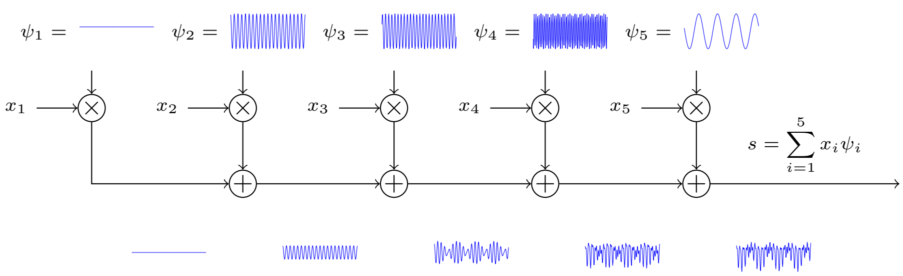
Dimensionality Reduction

- Can we reduce the burden from N to S early and often in the data processing pipeline?

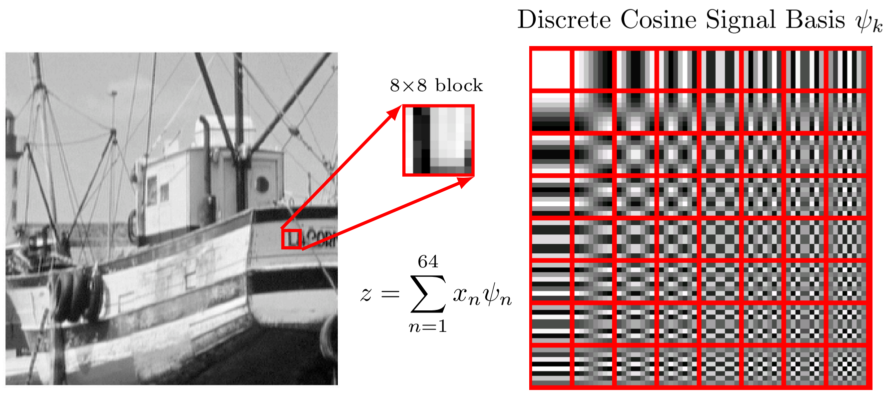
2.3 Signal Representation

Signal Representation: Signal Basis

- A signal basis can be used to define the class of signals of interest
- Example: represent a signal $z =$  as sum of scaled sinusoids



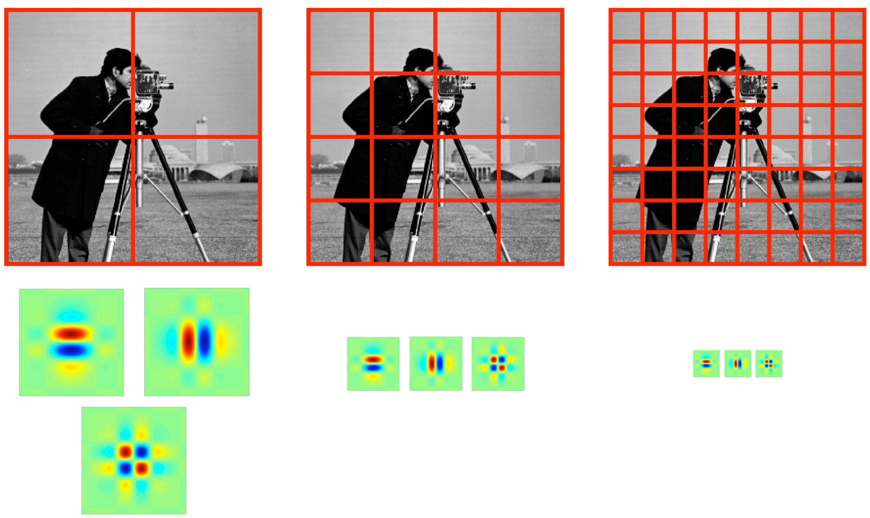
Lossy Compression: JPEG



credit: J. Romberg

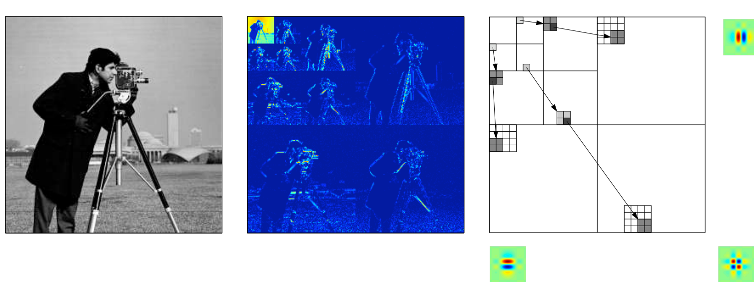
- Approximation with quantized coefs: $\hat{z} = \sum_{n=1}^{64} \hat{x}_n \psi_n$;

Multi-Scale Basis: Wavelets



credit: J. Romberg

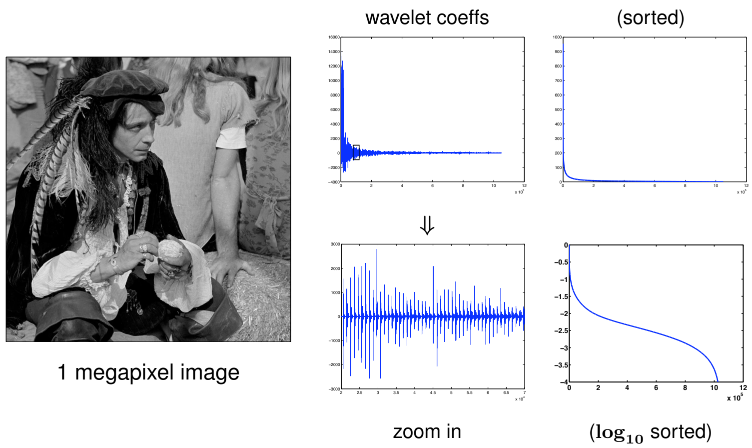
Wavelet coefficient representation



credit: J. Romberg

- A few large coefficients, but many small coefficients.

How many coefficients are important?

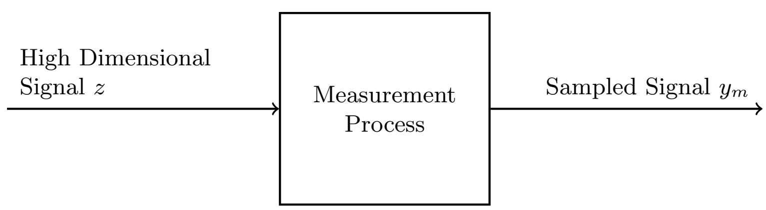


Conclusion

- Many classes of signals have a *sparse* representation in an appropriate basis

2.4 Measurement Models

Now let's bring in the measurement



Measurement Models

- Many measurement modalities are *linear*
- Inner product representation:

$$y_m = \langle z, \phi_m \rangle = \text{sum of point-wise product}$$

- Sampling

- Tomography

- MRI

credit: J. Romberg

Band-limited Signal Recovery: Signal and Measurement Model

- The signal/measurement model for the (1-D) Nyquist theorem uses
  - Signal model basis:
  $$\psi_n = e^{j\omega_0 nt}$$
  - Measurement: Sampling, M samples per period $T_s = T_0/M$.
  $$\phi_m = \delta(t - T_s m)$$
  Sampling frequency is $f_s = \frac{1}{T_s} = \frac{M}{T_0} = M\omega_0$.
  - *a priori* information: Band-limited signal, i.e. coefficients zero for $|n| \geq N_b$. Bandwidth: $\omega_b = N_b\omega_0$

Band-limited Signal Recovery: Set of Linear Equations

- Using this model,

$$y_m = \left\langle \sum_{n=-N_b}^{N_b} x_n \psi_n, \phi_m \right\rangle$$

$$y_m = \begin{bmatrix} & a'_m & \end{bmatrix} \begin{bmatrix} x_{-N_b} \\ x_{1-N_b} \\ \vdots \\ x_{N_b-1} \\ x_{N_b} \end{bmatrix}$$

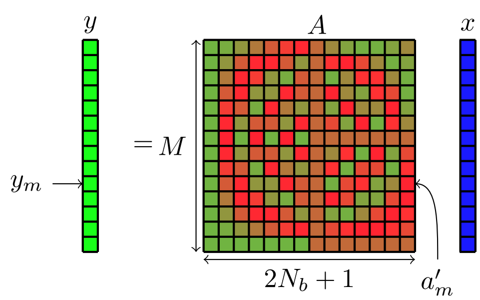
where $a_k = \begin{bmatrix} \langle \psi_{-N_b}, \phi_m \rangle & \cdots & \langle \psi_{N_b}, \phi_m \rangle \end{bmatrix}$

Band-limited Signal Recovery: Row Independence

$$\begin{aligned} a_k &= \begin{bmatrix} \langle \psi_{-N_b}, \phi_m \rangle & \cdots & \langle \psi_{N_b}, \phi_m \rangle \end{bmatrix} \\ &= \begin{bmatrix} e^{j\omega_0 T_s m(-N_b)} & e^{j\omega_0 T_s m(-N_b+1)} & \cdots & e^{j\omega_0 T_s m(N_b)} \end{bmatrix} \end{aligned}$$

- a_k looks like $e^{j\hat{\omega}n}$ with $\hat{\omega} = \omega_0 T_s m = \frac{2\pi}{M}$
- Orthogonality property of complex exponentials: a_i and a_j are orthogonal (and thus independent) for $0 < i \neq j \leq M$.

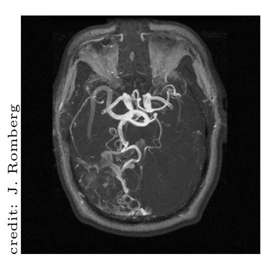
Band-limited Signal Recovery: Nyquist Recovery

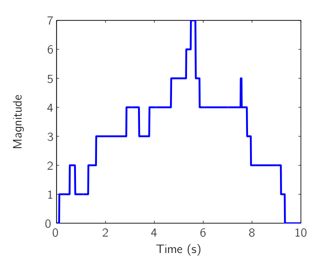


- Since rows are independent, need $M \geq 2N_b + 1$ to recover x.
- Implies $f_s \geq (2N_b + 1)\omega_0$: sampling frequency needs to be greater than two times bandwidth.

So what is the problem?

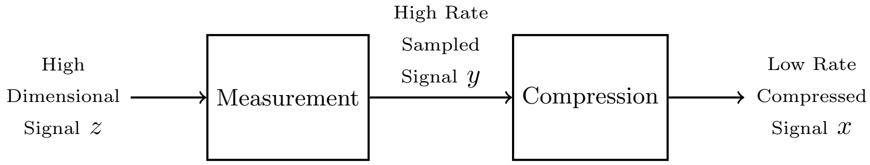
- Signals often have high bandwidth, but lower complexity content





- What if we change the signal model: not bandlimited, but sparse in some basis.

Current Solution: Measure Then Compress



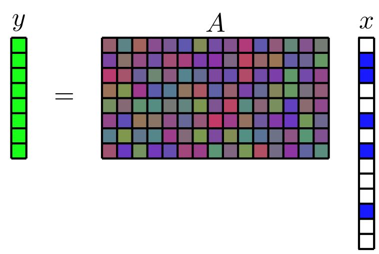
- Measurement costs \$. Compression costs \$.
- Can we combine the measurement and compression steps? (Compressive Sensing)

2.5 Sparse Signal Models

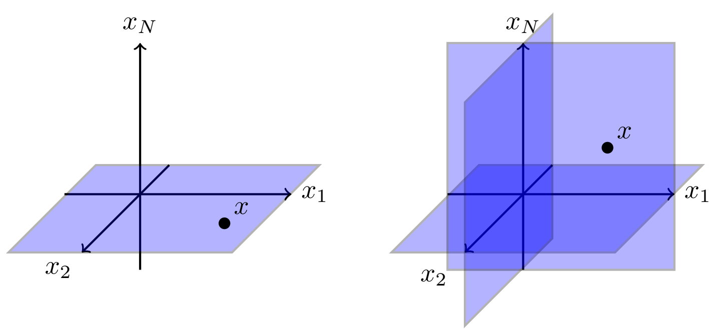
Sparse Signal Recovery: Compressive Measurement Model

- Model: signal $x \in \mathbb{R}^N$, with S-sparse support, measurement, $y \in \mathbb{R}^M$.
  - Ψ - signal basis (columns are ψ_n)
  - Φ - measurement matrix (rows are ϕ_m)

$$y = \underbrace{\Phi\Psi}_{A} x$$



Geometry of Signal Models



Linear Subspace, dim N_b
Bandlimited Signals

Union of dim S Subspaces
Sparse Signals

Sparse Signal Recovery: Recovery via regularization

- Given y, can we recover x?
- A is short and fat: non-trival null space means many solutions to $y = Ax$.
- Idea: regularized recovery

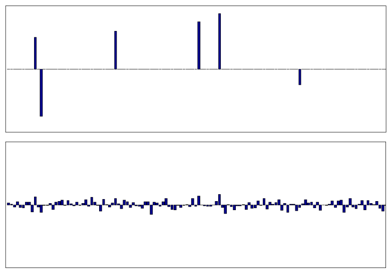
$$\hat{x} = \arg\min_x ||x||_* \quad \text{s.t. } y = Ax$$

Sparse Signal Recovery: ℓ_2 recovery

- ℓ_2-recovery (Euclidian distance) doesn't work

$$\hat{x} = \arg\min_x ||x||_2 \quad \text{s.t. } y = Ax$$

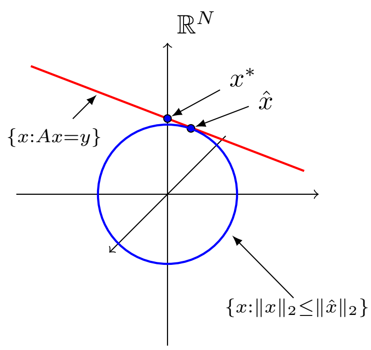
- Minimum is almost never sparse



x

$\hat{x} = (A'A)^{-1}A'y$

Sparse Signal Recovery: ℓ_2 recovery geometry



Incorrect Recovery

Sparse Signal Recovery: Sparcity preserving norms

- ℓ_0-recovery: $\|x\|_0 = \#$ of non-zero elements of x.

$$\hat{x} = \arg\min_x ||x||_0 \quad \text{s.t. } y = Ax$$

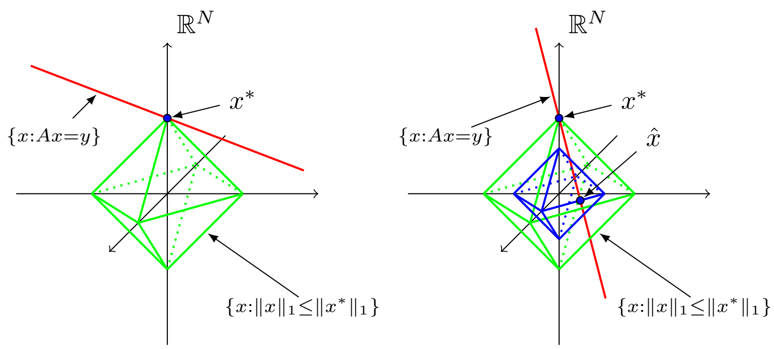
  - Works generically if $M = S + 1$. However, computationally demanding.

- ℓ_1-recovery: $\|x\| = \sum |x_i|$. Convex! Recovery via LP:

$$\hat{x} = \arg\min_x ||x||_1 \quad \text{s.t. } y = Ax$$

  - Also related to basis pursuit, lasso.
  - Works generically if $M \approx S \log N$!!!

Sparse Signal Recovery: ℓ_1 recovery geometry

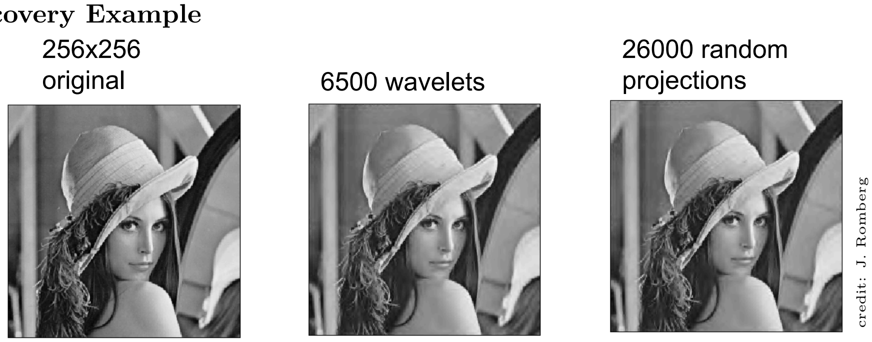


Correct Recovery   Incorrect Recovery

Other Recovery Methods

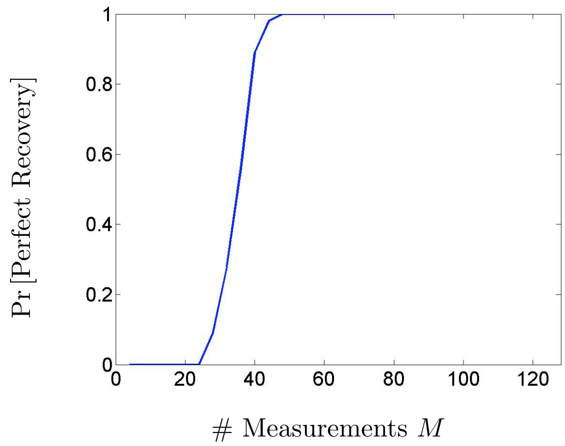
- Greedy methods - Orthogonal Matching Pursuit (Tropp, 2004)
- Iterative convex - Reweighted ℓ_1 - (Candès, Wakin and Boyd, 2008)
- Non-convex - smoothed ℓ_0 - (Chartrand, 2007; Mohimani et al., 2007)

Recovery Example



- Wavelets: 6500 largest coefficients
- 26000 random projections: recovery using wavelet basis
- Good approximation with 4x sampling rate over perfect knowledge

Recovery Example



Signal length $N = 128$, $S = 10$

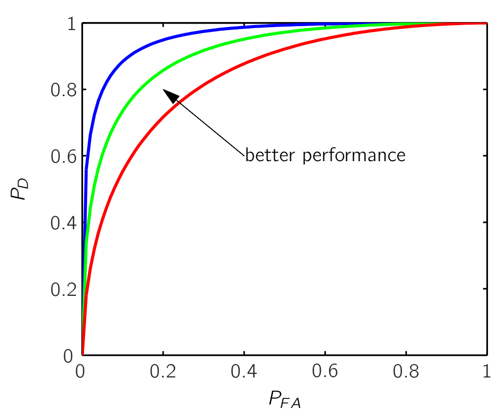
Sparse Signal Detection

- The classic signal detection problem:
  - known signal z may or may not have been sent
  - measurement y corrupted by noise v
- Define events $\mathcal{E}_0$ and $\mathcal{E}_1$ as:

$$\mathcal{E}_0 \triangleq y = v$$
$$\mathcal{E}_1 \triangleq y = z + v$$

- Detection algorithm: decide if event $\mathcal{E}_0$ or $\mathcal{E}_1$ occurred.
- Performance metrics are
  - false-alarm probability - $P_{FA} = \Pr\left[(\mathcal{E}_1 \text{ chosen when } \mathcal{E}_0)\right]$
  - detection probability - $P_D = \Pr\left[(\mathcal{E}_1 \text{ chosen when } \mathcal{E}_1)\right]$

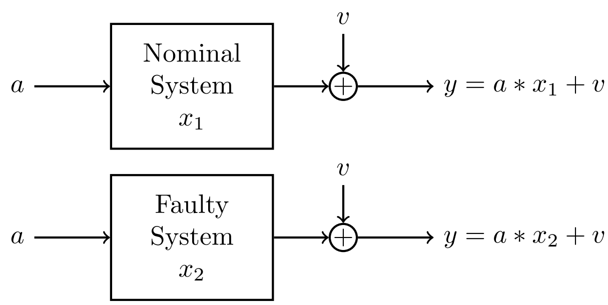
Receiver Operation Characteristic: ROC curve



- How many measurements are necessary to obtain the desired performance?

Fault Isolation

- Application: System with known fault condition. All signals are discrete time sequences.



- Subtract expected output: detection problem with $z = a * (x_1 - x_2)$
- Convolution: $z = A(x_1 - x_2)$, A Toeplitz Matrix.

Compressive Signal Processing

- Experiment model
$$y = \Phi\Psi x + v$$
  - Ψ - signal basis (columns are ψ_n)
  - Φ - measurement matrix (rows are ϕ_m)
  - $y \in \mathbb{R}^M$, measurement, $x \in \mathbb{R}^N$, S-sparse signal, $v \in \mathbb{R}^M$ measurement noise.
- Basic problems
  - Compressive recovery of unknown S-sparse signal using M measurements, with $S < M \ll N$.
  - Detection of a known S-sparse signal using M measurements, with $S < M \ll N$.

Compressive Signal Processing: Questions

- What are the conditions that guarantee that all x of a given sparsity can be recovered?
- What are the conditions that guarantee a particular level of performance in detection?
- How can we generate measurement matrices that meet these conditions?

3 Sparse Recovery

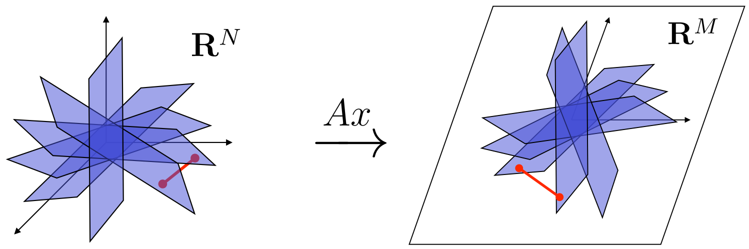
3.1 Sufficient Condition for Recovery: RIP

The Restricted Isometry Property (RIP)

- Introduced by Candès and Tao

Definition 1. X satisfies the RIP of order S if there exists a $\delta_S \in (0,1)$ such that
$$(1 - \delta_S)\,\|a\|_2^2 \le \|Xa\|_2^2 \le (1 + \delta_S)\,\|a\|_2^2$$
holds for all S-sparse signals a.

RIP as embedding



- Difference of two S-sparse signals is $2S$ sparse.

$$(1-\delta_{2S})\,\|u-v\|_2^2 \le \|A(u-v)\|_2^2 \le (1+\delta_{2S})\,\|u-v\|_2^2$$

Recovery Result: Candès (2008)

- Recovery algorithm (basis pursuit de-noising)

$$\hat{x} = \arg\min_x ||x||_1 \quad \text{s.t. } \|y - Ax\|_2 \le \epsilon$$

Theorem 2. *Suppose y is generated by $y = Ax^* + v$. If A satisfies RIP with $\delta_{2S} < \sqrt{2} - 1$ and $\|v\|_2 < \epsilon$, then*

$$\|\hat{x} - x^*\|_2 \le C_0 \frac{\|x^* - x_s\|}{\sqrt{s}} + C_1 \epsilon$$

where x_s is the S-sparse approximation of x^*.

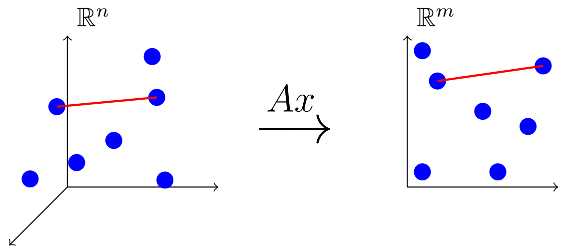
- Implies perfect recovery if x^* is S-sparse and no noise.

3.2 Generating Measurements That Satisfy RIP

Checking RIP

- Given A, does it satisfy RIP?
  - Check eigenvalues of each $M \times S$ submatrix - combinatorial.
- Generate A *randomly* - satisfies RIP with high probability when $M = O(S \log N)$!
  - iid Gaussian entries
  - iid Bernoulli entries (+/- 1)
  - random Fourier ensemble
  - (Candes, Tao; Donoho; Traub, Wozniakowski; Litvak et al)
- Proofs bound eigenvalues of random matrices, but generally difficult to generalize to $\Psi \neq I$.

Recall Johnson-Lindenstrauss Embedding



J-L Embedding
Given $\epsilon > 0$ and set $\mathbb{P}$ of P points in $\mathbb{R}^N$, find A such that for all $u, v \in \mathbb{P}$,

$$(1-\epsilon)\|u-v\|^2 \leq \|A(u-v)\|^2 \leq (1+\epsilon)\|u-v\|^2$$

Random J-L Embeddings

- Using our results from the last talk, we have the following:

Theorem 3 (Dasgupta and Gupta; Frankl; Achioptas; Indyk and Motwani)**.** *Given set $\mathbb{P}$ of P points in $\mathbb{R}^N$, choose $\epsilon > 0$ and $\beta > 0$. Let A be an $M \times N$ matrix with independent elements $[A]_{ij} \sim \mathcal{N}\left(0, \frac{1}{M}\right)$ where*

$$M \geq \left(\frac{7+6\beta}{\min(.5,\epsilon)^2}\right)\ln(P).$$

Then with probability greater than $1 - P^{-\beta}$, the following holds: For all $u, v \in \mathbb{P}$,

$$(1-\epsilon)\|u-v\|^2 \leq \|A(u-v)\|^2 \leq (1+\epsilon)\|u-v\|^2$$

Other Favorable Random Mappings: Sub-Gaussian Distributions

- In the proof, we used
$$[A]_{ij} \sim \mathcal{N}\left(0, \frac{1}{M}\right)$$
- Key step was Chernoff bound using moment generating function

Definition 4. A random variable X is *Sub-Gaussian* if there exists an $a \geq 0$ such that

$$\mathbb{E}\left[e^{sX}\right] \leq e^{\frac{a^2 s^2}{2}}$$

and τ, the smallest such a, is called the *Gaussian standard* of X.

Other Favorable Random Mappings: Properties of Sub-Gaussians

Key Properties

- If X_i are iid sub-Gaussian, $Y = \sum X_i$ is sub-Gaussian with standard $\tau_y \leq \sum \tau_{x_i}$
- If X is sub-Gaussian with standard τ, $\mathbb{E}\left[e^{sX^2}\right] \leq \frac{1}{1-2s\tau^2}$

Other Favorable Random Mappings: Sub-Gaussian Examples

- We can use any zero mean sub-Gaussian iid sequence with variance $1/M$.
- Rademacher Sequence

$$[A]_{ij} = \begin{cases} +\frac{1}{\sqrt{M}} & \text{with probability } \frac{1}{2} \\ -\frac{1}{\sqrt{M}} & \text{with probability } \frac{1}{2} \end{cases}$$

- "Database-friendly" (Achlioptas)

$$[A]_{ij} = \begin{cases} +\sqrt{\frac{3}{M}} & \text{with probability } \frac{1}{6} \\ 0 & \text{with probability } \frac{1}{3} \\ -\sqrt{\frac{3}{M}} & \text{with probability } \frac{1}{6} \end{cases}$$

From JL to RIP

- Baraniuk et al. (2008)
- Consider measurement with $\Psi = I$, Φ random elements from a favorable distribution

$$y = \underbrace{\Phi\Psi}_{A} x$$

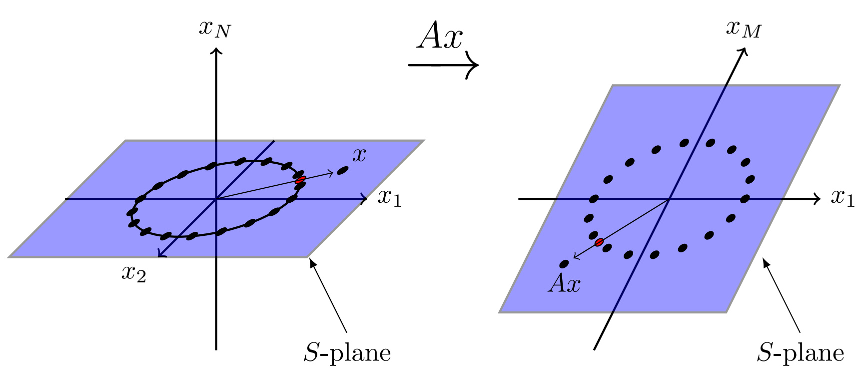
- Favorable distribution implies that for given $x \in \mathbb{R}^N$,

$$\Pr\left[\left|\|Ax\|_2^2 - \|x\|_2^2\right| \geq \epsilon\|x\|_2^2\right] \leq 2e^{-Mc_0(\epsilon)}$$

- pick $\epsilon = \delta_{2S}/2$

From JL to RIP

- Examine mapping on one of $\binom{N}{S}$ S-planes in sparse model
  - Construct (careful) covering of unit sphere using $(12/\delta_{2S})^S$ points
  - JL: isometry for each point with high probability
  - Union bound for all points
  - Extend isometry to all x in unit ball (and thus all x in S-plane)



A look at the probabilities: Union Bounds

- Probability of error $> \frac{\delta_{2S}}{2}$ when mapping 1 point
$$\leq 2e^{-Mc_0(\delta_{2S}/2)}$$
- Probability of error when $(12/\delta_{2S})^S$ points mapped
$$\leq 2(12/\delta_{2S})^S e^{-Mc_0(\delta_{2S}/2)}$$
- "Careful" covering implies that for all x in unit ball, $\exists q$ in covering s.t. $\|x-q\| < \delta_{2S}/4$.
- Probability of error $> \delta_{2S}$ when unit ball mapped
$$\leq 2(12/\delta_{2S})^S e^{-Mc_0(\delta_{2S}/2)}$$

A look at the probabilities, continued

- Probability of error $> \delta_{2S}$ when $\binom{N}{S}$ planes mapped:
$$\leq 2\binom{N}{S}(12/\delta_{2S})^k e^{-Mc_0(\delta_{2S}/2)} \leq 2e^{-c_0(\delta_{2S})M+S[\ln(eN/S)+\ln(12/\delta_{2S})]}$$
using bound $\binom{N}{S} \leq (eN/S)^S$.

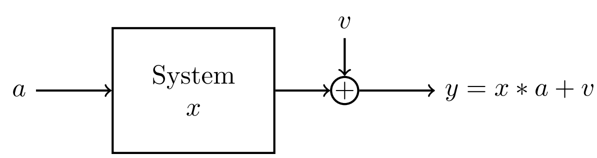
Result
If $M > O(S\log(N/S))$, with probability greater than $1-2e^{-c_2M}$, A random matrix with favorable distribution satisfies RIP.

- Bonus: Universality for orthonormal basis Ψ: only changes orientation of planes in model.

4 Structured Compressive Signal Processing

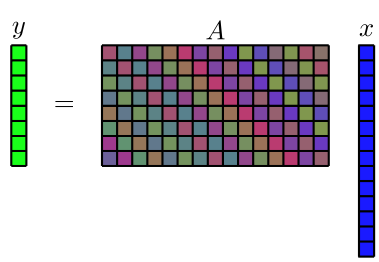
Structured Measurements: A Detection Problem with Convolution

- We are not always free to choose the elements of Φ independently
  - Distributed measurements
  - Dynamic Systems



Convolution implies Toeplitz measurement matrix

$$y = a * x$$



- Cannot choose the elements of A independently

Concentration of Measure for Toeplitz matrices

- Suppose a is chosen iid Gaussian $x_i \sim \mathcal{N}\left(0, \frac{1}{M}\right)$.
- For fixed x, $y \sim \mathcal{N}\left(0, \frac{1}{M}P\right)$ where

$$[P]_{ij} = \sum_{i=1}^{n-|i-j|} x_i x_{i+|i-j|}$$

- Let $\rho(x) = \frac{\lambda_{\max}(P)}{\|x\|_2^2}$ and $\mu(x) = \frac{\frac{1}{d}\sum \lambda_i^2(P)}{\|x\|_2^2}$.

Result
For any $\epsilon \in (0, 0.5)$

$$\Pr\left[\|Ax\|_2^2 \geq \|x\|_2^2 (1+\epsilon)\right] \leq e^{-\epsilon^2 M/6\rho(a)}$$
$$\Pr\left[\|Ax\|_2^2 \leq \|x\|_2^2 (1-\epsilon)\right] \leq e^{-\epsilon^2 M/4\mu(a)}$$

Sanandaji, Vincent, Wakin - 2010 Conference on Decision and Control

Implications

- Recall result from previous lecture for A unstructured:

$$\Pr\left[\|Ax\|_2^2 \geq \|x\|_2^2 (1+\epsilon)\right] \leq e^{-\epsilon^2 M/6}$$
$$\Pr\left[\|Ax\|_2^2 \leq \|x\|_2^2 (1-\epsilon)\right] \leq e^{-\epsilon^2 M/4}$$

- Concentration bound worsens over i.i.d. entries by factors ρ and μ.
- Bound: $\mu(a) \leq \rho(a) \leq \|a\|_0$. However, most a are must less than this bound.

Fault Detection Problem

- System impulse response can be x_1 or x_2.
- record $\tilde{y} = y - Ax_1$, let $\delta x = x_2 - x_1$
- Define events $\mathcal{E}_0$ and $\mathcal{E}_1$ as:

$$\mathcal{E}_0 \triangleq \tilde{y} = v$$
$$\mathcal{E}_1 \triangleq \tilde{y} = A\delta x + v$$

- Detection algorithm: decide if event $\mathcal{E}_0$ or $\mathcal{E}_1$ occurred.
- Performance metrics are
  - false-alarm probability - $P_{FA} = \Pr\left[(\mathcal{E}_1 \quad \text{chosen when} \quad \mathcal{E}_0)\right]$
  - detection probability - $P_D = \Pr\left[(\mathcal{E}_1 \quad \text{chosen when} \quad \mathcal{E}_1)\right]$

Neyman-Pearson Test

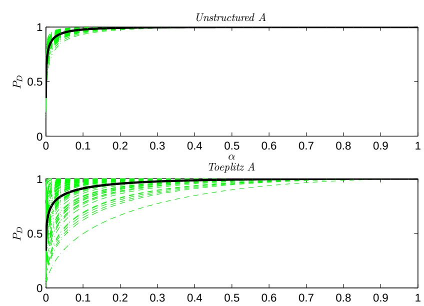
- The Neyman-Pearson detector maximizes P_D for a given limit on failure probability, $P_{FA} \leq \alpha$ under Gaussian noise assumption.
$$\tilde{y}' Ax \gtrless_{\mathcal{E}_0}^{\mathcal{E}_1} \gamma$$

- Performance:
$$P_D = Q\left(Q^{-1}(P_{FA}) - \frac{\|Ax\|_2}{\sigma}\right)$$

- Since performance depends on $\|Ax\|_2$, worse performance for signals with large $\rho(a)$, $\mu(a)$.

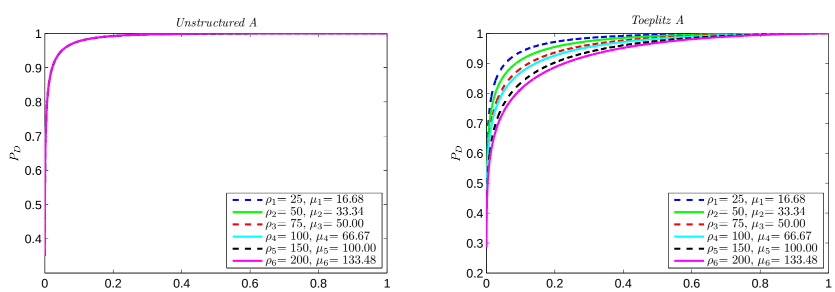
Detection Performance

- $y = Ax + v$
- A is 125×250
- x is block sparse, $\mu(a) = 33$, $\rho(a) = 50$.
- Two cases:
  - A - Unstructured
  - A - Toeplitz
  - 1000 realizations of A



Detection Performance

- Average detection performance for six different x.



Conclusion

- Compressive Sensing - going beyond Nyquist sampling
- Sparse signal model with linear measurement model
- Recovery possible using convex optimization
- Work continues on
  - Recovery methods
  - Structured measurements
  - New applications - development of sparse signal models
  - ...