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Lecture 1

Quickest detection & the problem of two-sided alternatives

The topic of statistical surveillance is presented. We begin by defining the out-of-control and in-control states of the process and describe how we can distinguish them by using statistics based on the observations of the process. In this context we present the CUSUM statistic. As an example of a problem in statistical surveillance we will discuss the topic of change-point detection in the Brownian motion model with multiple alternatives. In this problem, the objective is to detect a change in the constant drift by means of a stopping rule that reacts fairly fast to an abrupt change but at the same time keeps the frequency of false alarms below a certain threshold. In this context we also examine the specific instance of two-sided alternatives, in the context of sequential detection of an abrupt change in the drift of a Brownian motion in the presence of two-sided alternatives. In particular, we derive the first moment of a general 2-CUSUM stopping rule in closed-form under all relevant measures. We form a min-max stochastic optimization problem using an extended Lorden criterion and assert that the optimal solution lies in the class of equalizer rules. We then compare the performance of the general classical 2-CUSUM equalizer stopping rules to the performance of the modified drift 2-CUSUM harmonic mean equalizer rules whose performance is also available in closed-form. It is seen that in the case of a symmetric change the modified drift 2-CUSUM harmonic mean equalizer rules display a slightly better performance to the performance of the classical 2-CUSUM stopping rules for small values of the mean time between false alarms and small values of the change in the drift. In the non-symmetric case however, it is seen that the classical 2-CUSUM equalizer rules display in general superior performance to the modified drift 2-CUSUM harmonic mean equalizer rules. We finally discuss the asymptotic properties of all of the above 2-CUSUM stopping rules.

Lecture 2

Multi-dimensional quickest detection

We consider the problem of quickest detection in the presence of multiple random sources each driven by distinct sources of noise represented by a Brownian motion. We make the assumption that the driving noises are independent. We identify the problem of two-sided alternatives as a special case of this problem in the case that the driving Brownian motions have a correlation equal to -1. The case described in this set-up corresponds to 0 correlation in the noise component of each source. The first problem we will address is the one of detecting a change in the drift of Brownian motions received in parallel at the sensors of decentralized systems. We examine the performance of one shot schemes in decentralized detection in the case of many sensors with respect to appropriate criteria. One shot schemes are schemes in which the sensors communicate with the fusion center only once; when they must signal a detection. The communication is clearly asynchronous and we consider the case that the fusion center employs one of two strategies, the minimal and the maximal. According to the former strategy an alarm is issued at the fusion center the moment in which the first one of the sensors issues an alarm, whereas according to the latter strategy an alarm is issued when both sensors have reported a detection. In this work we derive closed form expressions for the expected delay of both the minimal and the maximal strategies in the case that CUSUM stopping rules are employed by the sensors. We prove asymptotic optimality of the above strategies in the case of across-sensor independence and specify the optimal threshold selection at the sensors. Moreover, we consider the problem of quickest detection of signals in a coupled system of N sensors, which receive continuous sequential observations from the environment. It is assumed that the signals, which are modeled by a general Ito processes, are coupled across sensors, but that their onset times may differ from sensor to sensor. Two main cases are considered; in the first one signal strengths are the same across sensors while in the second one they differ by a constant. The objective is the optimal detection of the first time at which any sensor in the system receives a signal. The problem is formulated as a stochastic optimization problem in which an extended minimal Kullback-Leibler divergence criterion is used as a measure of detection delay, with a constraint on the mean time to the first false alarm. The case in which the sensors employ cumulative sum (CUSUM) strategies is considered, and it is proved that the minimum of N CUSUMs is asymptotically optimal as the mean time between false alarms increases without bound. In particular, in the case of equal signal strengths across sensors, it is seen that the difference in detection delay of the N-CUSUM stopping rule and the unknown optimal stopping scheme tends to a constant related to the number of sensors as the mean time between false alarms increases without bound. While in the case of unequal signal strengths, it is seen that this difference tends to 0.

Lecture 3

Drawdowns and drawups, their joint distributions and connections to the 2-CUSUM, detection and financial risk management.

In this lecture we will consider drawdowns and drawups under various models, beginning with the biased random walk, the drifted Brownian motion and continuous-time diffusions. The topic of interest will be to derive the probability that a drawup of a unit precedes a drawdown of equal units in the presence of a finite time horizon. The drawup process is defined as the difference of the present value of the holdings of an investor and its historical minimum, while the drawdown process is defined as the difference of the historical maximum and its present value. The first hitting time of these processes to certain thresholds b and a are known as the drawup and the drawdown respectively. By drawing connections of joint probabilities involving the drawdowns and the drawups of equal units to the range process we will derive quantities related to their joint distribution. In particular we derive the Laplace transform of the drawdown when it precedes the drawup under general diffusion dynamics and discuss how this can lead to the computation of the probability in question. In the case of unequal units of drawdowns and drawups, we will derive the derivation of the Laplace transform in question sis achieved through path decomposition and the use of the Markov property. We will then proceed to draw connections of the drawdown and the drawdown to the 2-CUSUM and to the way in which understanding the joint probabilistic properties of the drawdowns and the drawups can help us address the problem of transient signal detection especially in the case in which the life-time of the signal is an exponentially distributed random variable independent of the underlying signal. We finally identify the drawup and the drawdown as measures of relative satisfaction and regret of an investor and recognize the usefulness of our results in the valuation of digital options whose payoff depends on drawdowns and drawups.

Lecture 4

Financial risk managent: Insuring Against Maximum Drawdown and Drawing Down Before Drawing Up

In this lecture we use results on the joint distribution of drawdowns and drawups to derive static hedges of digital options whose payoff depends on them. The drawdown of an asset is a risk measure defined in terms of the

running maximum of the asset's spot price over some period [0,T]. The asset price is said to have drawn down by at least $K over some period [0,T] if there exists a time below its maximum-to-date. Similarly, the drawup of an asset is a performance measure defined in terms of the running minimum of the asset's spot price over some period [0,T]. The asset price is said to have drawn up by at least $K over some period [0,T] if there exists a time$ when the stock price was at least $K above its minimum-to-date. We introduce two digital options whose payoff insures against the adverse movements in market. A digital call written on the maximum drawdown pays $1 at its maturity date T if and only if the spot price has drawn down by at least $K over the period [0,T]. In contrast, a digital option betting on drawing down before drawing up pays $1 at its maturity date its maturity date T if and only if the spot price draws down by at least K>0 before it draws up by $K. The buyer of the former claim is clearly insuring against maximum drawdown. The buyer of the latter claim is insuring against the event that a drawdown of at least $K before T occurs before a drawup of the same size. In this work we present model-free robust static hedges of the latter claim using one-touch knockouts and their spreads. We also provide semi-static hedge of the former claim using double barrier options. Since these instruments are relatively illiquid at present, we also derive semi-static hedges using single barrier one-touches and vanilla options under symmetry and continuity assumptions. Finally, we develop semi-static hedges of the target digital options under geometric symmetry.