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Concentration of measure: fundamentals and tools

This talk provides a tutorial covering basic material on concentration inequalities of functions of independent random variables around their mean. We will start with the inequalities of Markov, Chernoff and Hoeffding and end with the logarithmic Sobolev inequalitis of Ledoux and Talagrand's convex distance inequality.

We will also discuss other inequalities that apply to Gaussian processes. The focus will be on inequalities that play a role in applications to signal processing and compressive sensing and thus we will provide examples that show the practical use of the results.

Applications of concentration of measure in signal processing

This talk presents applications of concentration of measure phenomena in the emerging field of Compressive Sensing (CS). CS builds on the premise that a signal having a sparse representation in some basis can be recovered from a small number of linear measurements of that signal. Many of the most effective constructions for the linear measurement operator involve random matrices, and at the heart of much analysis in CS is a precise statistical characterization of the product of a random matrix with a sparse signal. Building on a simple concentration of measure inequality, for example, it is possible to generalize the Johnson-Lindenstrauss lemma and ensure an approximate distance-preserving embedding for an entire family of sparse signals. This "Restricted Isometry Property" for the measurement operator has been shown in CS to permit stable recovery of sparse signals from small numbers of measurements. We will also discuss a related problems in Compressive Signal Processing (CSP), in which the goal is not to recover (high-complexity question) a sparse signal (low-dimensional model) but rather to answer low-complexity questions about arbitrary high-dimensional signals. We will discuss how concentration of measure inequalities can be used to give performance guarantees for problems such as compressive detection and estimation.