ESTIMATION OF SURVIVAL CURVES UNDER UNIFORM STOCHASTIC ORDERING CONSTRAINT

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Abstract

In this paper we will discuss an estimator of the survival curve for F under the constraint of uniform stochastic ordering, where F and G are cumulative distribution functions (cdf) and G is fixed and known. As defined in Rojo and Samaneigo's paper, suppose F and G are cumulative distributions functions on $[0, \infty)$, and $\bar{F}$ and $\bar{G}$ are their corresponding survival functions, it follows that F is said to be uniformly stochastically smaller than G, written $F <_{(+)} G$ if and only if the assumption that the corresponding failure rates, h_G and h_F, are ordered is true when both F and G are absolutely continuous. Also, the uniform stochastic ordering implies the ratio $l(x) = \bar{G}(x)/\bar{F}(x)$ is nondecreasing for $x \in [0, \sup\{t : \bar{F}(t) > 0\})$. Research for the survival curve estimation will be considered using estimated hazard rates of the form $\hat{h}(x) = \hat{f}(x)/(1 - \hat{F}(x))$. Using known methods of density estimation, a data-based hazard estimate $\hat{h}_{F_n}$ will be computed, and given that $h_G \leq h_F$ an estimator $\hat{h}_F$ for h_F is given and shown to be a projection of $\hat{h}_{F_n}$ onto the set of hazard rates $\{h : h < h_G\}$. From there, an estimator for $\bar{F}$ will be proposed. We found through heuristic reasoning that this method will not generate accurate results when compared with previously used estimators, such as the empirical survival curve $\bar{F}$ and the Rojo-Samaniego estimator.