1. **Gibbs sampler.** Consider the file `school1.dat` (from the Book) which contains data on the amount of times students at a high school spent on studying or homework during an exam period. Suppose we fit the following model:

\[
Y_i \sim \mathcal{N}(\mu, \sigma^2), \quad i = 1, \ldots, n
\]

\[
\mu \sim \mathcal{N}(\mu_0, \tau_0^2)
\]

\[
\sigma^2 \sim \text{Inv-Gamma}(\alpha, \beta)
\]

with \( \mu_0 = 5, \tau_0^2 = 0.5, \alpha = 1, \beta = 4. 

(a) Evaluate the full conditionals, \( p(\mu | \sigma^2, y) \) and \( p(\sigma^2 | \mu, y) \).

(b) Implement the Gibbs sampler to obtain draws from the joint posterior \( p(\mu, \sigma^2 | y) \).

(c) Graph a density estimate for \( p(\sigma^2 | y) \) based on your MCMC output. That is, after removing an appropriate burn-in, plot a histogram of the sampled \( \sigma^2 \) and/or the kernel density estimate.

(d) Evaluate the marginal posterior density \( p(\sigma^2 | y) \) analytically up to a normalizing constant.

2. **Metropolis algorithm** Implement a Metropolis-Hastings on the same data but fixing \( \sigma^2 = 1 \).

(a) Consider a proposal function for \( \mu \) centered at the previously sampled value. Consider two different values for the variance proposal, \( \sigma_p = 0.5 \) and \( \sigma_p = 2 \).

(b) Provide a trace plot and a plot of the kernel density estimate for each chain.

(c) After removing an appropriate burn-in, evaluate the acceptance rate and the autocorrelation for varying lags. Which value of \( \sigma_p \) provides a better choice for the proposal function?

3. Hoff – Exercise 6.2

4. Hoff – Exercise 10.2