

The following are a list of symbols used in "Multitype infinite-allele branching processes in continuous time" by T.O. McDonald and M. Kimmel.

Notation for Markov Process (Section 2)

$\alpha_{i,j}(t)$	Frequency spectrum is the number of i -type labels having j individuals alive at time t
$\beta_r(t)$	$E_r \left[\sum_{n=1}^{N(t)} \boldsymbol{\rho}_n \mathbf{M} \mathbf{q}_{i,j}(t - T_n)^\top \right]$
$\delta_{n,w}$	Indicator that the n^{th} split was from a type w individual
λ	Eigenvalue with maximum real part of \mathbf{A}
$\tilde{\mathbf{Z}}(t)$	k -type branching process that counts the number of individuals alive at time t with the ancestral label
\mathbf{e}_i	Unit vector with zeros in all entries and 1 at the i^{th} element
$\mathbf{q}_{i,j}(t)$	$(q_{1,i,j}(t), \dots, q_{k,i,j}(t))$
\mathbf{u}	Right eigenvector of \mathbf{A} associated with the eigenvalue λ
\mathbf{v}	Left eigenvector of \mathbf{A} associated with the eigenvalue λ
$\mathbf{Z}(t)$	k -type branching process that counts the number of individuals alive at time t
\mathbf{A}	Infinitesimal generator of the mean process, $\mathbf{A} = D_{\mathbf{a}}(\mathbf{M} - \mathbf{I})$
$D_{\mathbf{a}}$	$\text{diag}(\mathbf{a})$
$\mathbf{M}(t)$	Mean offspring matrix of $\mathbf{Z}(t)$
\mathbf{M}	Mean offspring matrix containing entries m_{ij}
ν	Probability that an offspring is a new label
$\phi_{i,j}(t)$	Expectation of the frequency spectrum, $\phi_{i,j}(t) = E[\alpha_{i,j}(t)]$
$\rho_{n,w}$	$P(\delta_{n,w} = 1)$
$\tilde{q}_{r\mathbf{j}}(t)$	Probability of a type r ancestor having \mathbf{j} descendants at time t
a_i	The rate parameter for the lifetime distribution of a type i individual
$A_i(\mathbf{s}; t)$	Ancestral p.g.f. for $\tilde{Z}_i(t)$

$E_r[\cdot]$	Expectation given a single r -type ancestor
$f_i(\mathbf{s})$	Offspring p.g.f. for a type i individual
$H_i(\mathbf{s})$	Ancestral label offspring p.g.f. for a type i individual
$I_{0,r,i,j}(t)$	\mathbf{I} (the ancestor is type r and has j type i descendants with the ancestral label at time t)
$I_{n,m,l,i,j}(t)$	\mathbf{I} (the m^{th} l -type individual born at time T_n acquires a new label and has j i -type descendants with that same label at time t)
$K_i(t)$	Number of type i labels with individuals alive at time t
m_{ij}	Mean number of type j offspring from type i parent
$N(t)$	Number of splits in $(0, t]$
$q_{r,i,j}(t)$	Probability of a type r ancestor having j type i descendants at time t
T_n	The n^{th} splitting time of $\mathbf{Z}(t)$
$U_{n,i}$	Number of type i offspring from the n^{th} split in the process

Notation for General Branching Process (Section 3)

0	The ancestor of the population
α	Malthusian parameter
$\alpha_{i,\Gamma}(t)$	Frequency spectrum for a set, Γ
*	Composition operation consisting of a transition on the state space and convolution on \mathbb{R}^+
$\bar{\xi}(t)$	$\int_{S \times \mathbb{R}^+} e^{-\alpha t} h(s) \xi(ds \times dt)$
β	Mean age at progeny production
$\check{\mu}(r, A \times B)$	Reproduction kernel for offspring with new labels
$\check{\xi}(t)$	Point process of progeny with a new label
$\chi_{\mathbf{x}}(a)$	A random characteristic for individual \mathbf{x}
$\gamma(k, \omega)$	Indicator that the k^{th} daughter of an individual with life history ω has the same label as its parent
$\hat{g}_\alpha = \hat{g}(\alpha)$	$\int_{\mathbb{R}^+} e^{-\alpha t} g(dt)$

$\hat{q}(r, i, \alpha)$	Laplace transform of $q_{r,s,0}(t)$
$\mathbf{x} = (x_1, \dots, x_n)$	An individual in the population; the x_n^{th} daughter of the x_{n-1}^{th} daughter of the ... of the x_1^{th} daughter of the ancestor
$\mathbf{x}_{[k]}$	the k^{th} ancestor of \mathbf{x}
\mathcal{S}	σ -algebra generated by Ω
$\mu(r, A \times B)$	Reproduction kernel, or expectation of $\xi(A \times B)$ at time t
$\omega = \omega_{\mathbf{x}}$	Life history of an individual \mathbf{x}
Ω	Set of all possible life histories
$\phi_{i,\Gamma}(t)$	Expectation of $\alpha_{i,\Gamma}(t)$
$\pi(A)$	Eigenmeasure for $\hat{\mu}_\alpha(r, ds)$, $\pi(A) = \int_{\mathcal{S}} \hat{\mu}_\alpha(r, A) \pi(dr)$
Π_r	The probability measure for the life history associated with a type r individual
ψ	Life length of an individual
$\rho(k, \omega)$	Type of the k^{th} daughter of an individual with life history $\omega \in \Omega$
$\sigma_{\mathbf{x}}$	Birth time of \mathbf{x}
$\tau(k, \omega)$	Age of an individual with life history $\omega \in \Omega$ at the time of birth of its k^{th} daughter
$\tilde{\mu}(r, A \times B)$	Reproduction kernel for offspring with the same label as the parent
$\tilde{\xi}(t)$	Point process of progeny with the parent label
$\tilde{q}_{r\mathbf{0}}(t)$	Probability of extinction of $\tilde{\mathbf{Z}}(t)$ given a type r ancestor
$\tilde{Z}_s(t)$	Number of s -type individuals alive at time t with the ancestral label
$\xi(A \times B, \omega) = \xi(t)$	Reproduction process of an individual with life history ω with $\rho(i, \omega) \in A$ and $\tau(i, \omega) \in B$
$\{Y(t) \rightarrow \infty\}$	Nonextinction set, or set of processes that do not ever go extinct
$g_\alpha(u)$	$e^{-\alpha t} g(u)$
$h(r)$	Eigenfunction for $\hat{\mu}_\alpha(r, ds)$, $\pi(r) = \int_{\mathcal{S}} h(s) \hat{\mu}_\alpha(r, ds)$

I	The set of all descendants of the population
$K_i(t)$	Number of type i labels represented by individuals alive at time t excluding the ancestral label
$n(\mathbf{x})$	The generation of \mathbf{x}
$N_i(t)$	Total number of type i labels ever existing up to time t excluding the ancestral label
$q_{r,s,0}(t)$	Probability that there are no type s individuals alive at time t given a type r ancestor
S	The type-space for individuals
$S_{\mathbf{x}}$	Shift operator that treats \mathbf{x} as an ancestor
w_t	Intrinsic martingale associated with the branching process
$Y(t)$	Number of births up to time t
$Z^\chi(t)$	A branching process counted by characteristic χ up to time t