

Stat 331/Elec 331, Homework 7, December 2

1. Consider the following queueing system: customers arrive in pairs according to a Poisson process with rate $\alpha = 1$ customer pair/minute. There is one server and room for two customers to wait in line. Service times are exponential with mean 1 minute. If there is not room for both arriving customers, they both leave.

a. Describe the system in a rate diagram and find the stationary distribution.

b. Suppose the system leaves state 2. Let T be the time until it is back in state 2 again. What is $E[T]$? (4)

2. Consider an $M/M/1$ queueing system in equilibrium. Let N be the number of customers in the system, Q the number of customers waiting in line, T the total time spent in the system for an arriving customer and W the waiting time until service for an arriving customer.

a. What is the relation between Q and N ?

b. What is the relation between T and W ?

c. Is W discrete? Continuous? Why/why not? Show in a figure what the cdf of W looks like (roughly, no equations needed). (4)

3. Phone calls arrive to a company according to two independent Poisson processes, one of female callers with rate 1 and one of male callers with rate 2 (calls/minute). There is one server and room for one to wait in line. If the server is busy, a female caller stays to wait in line with probability 0.8, a male caller with probability 0.5. Service times are i.i.d. exponential with mean length 2 minutes.

a. Describe the system in a rate diagram and find the stationary distribution.

b. What proportion of callers are lost? (4)