Final exam, Stat/Elec 331, Fall 2003

Solutions should be clear, complete and easy to follow. You are allowed to use the book, a calculator, your lecture notes, and lecture notes posted on the course web page. The time limit is five hours. Each problem is worth 6 points. The deadline to turn in the exam is December 17, 5 pm. Late turn-ins are not accepted.

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1. The continuous random variable X has pdf

$$f(x) = \begin{cases} 1/2 & \text{if } 0 < x \leq 1 \\ 1/(2x^2) & \text{if } x \geq 1 \end{cases}$$

a. Show that this is a possible pdf for a continuous random variable.

b. Compute $E[X]$.

c. Find the cdf of X and sketch its graph.

d. Let $Y = 1/X$. Find the cdf of Y and sketch its graph.

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2. A company manufactures metal plates of size 5 x 10 (inches). Due to random fluctuations, a manufactured plate has a size of X x Y inches where (X, Y) follows a bivariate normal distribution with means 5 and 10, variances 0.01 and 0.04 and correlation coefficient 0.8. Let C be the circumference (perimeter) and A the area of a plate

a. Find $E[C]$ and $E[C|X = x]$.

b. Find $E[A]$ and $E[A|X = x]$.

c. A plate is useful if $29 \leq C \leq 31$. What is the probability that a plate is useful?

d. Now suppose that you have ten plates. What is the probability that at least nine are useful?

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3. The random variable X has pdf

$$f(x) = (a+1)x^a, \quad 0 \leq x \leq 1$$

where a is an unknown parameter.

a. Find the maximum likelihood estimator and the moment estimator of a based on a sample $X_1, X_2, ..., X_n$.

b. Suppose $a = 2$. Describe how you can simulate observations on X based on observations from a uniform [0,1]-distribution. If such a uniform value is 0.008, what value of X does this give?

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4. Accidents on a certain road occur according to a Poisson process with rate λ accidents/week.

a. The two towing companies A and B have agreed to take turns in dealing with the accidents. Thus, A takes care of the first accident, B the second and so on. Consider the process of accidents that A takes care of. Is this a Poisson process? If so, what is the rate?

b. Suppose that in a particular year, it is observed that N of the 52 weeks had no accidents. What is the distribution of N (name and parameters)?

c. Based on N, find the moment estimator of λ. Note that we only have one observation on N, so our sample size is $n = 1$.

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5. Customer groups arrive to a service station according to a Poisson process with rate α groups/minute. With probability p, such a group consists of a single individual and with probability $1-p$ it consists of a pair. There is a single server and room for two to wait in line. Service times are exponential with rate β (mean $1/\beta$). If a pair arrives and they cannot both join, they both leave.

a. Give the state space and describe the system in a rate diagram.

b. Suppose $\alpha = \beta$ and $p = 1/2$. State the balance equations and find the stationary distribution $(\pi_0, \pi_1, \pi_2, \pi_3)$.

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6. Consider the plots on the next page. Each is a plot of 1000 simulated observations on a pair of random variables (X, Y). Which of them do you think come from bivariate normal distributions? For those that do, what can you say about the correlation coefficient ρ (negative, positive, zero, small, large etc)?

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7. Bonus question, not included in the time limits. As you may know, there is an element named after California, element number 98, californium. It's perhaps more interesting that as many as four elements are named after a small village in a northern European country. Which country, village and elements? No points for this questions, just the pleasure of acquiring obscure facts. Googling allowed.

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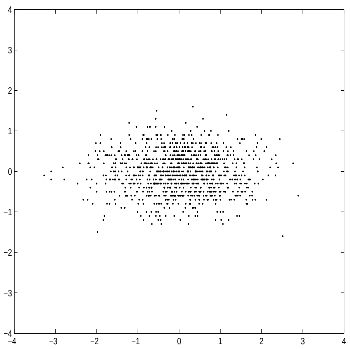
(a)



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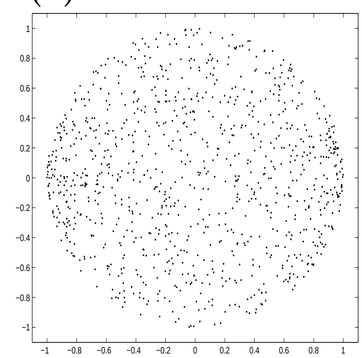

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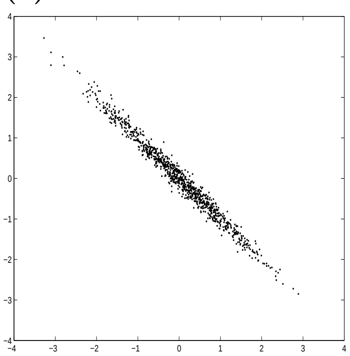

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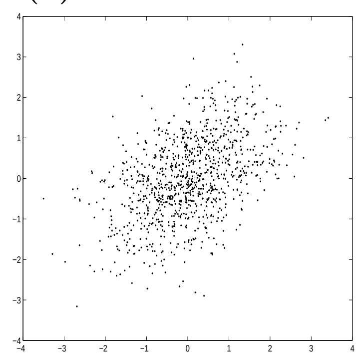

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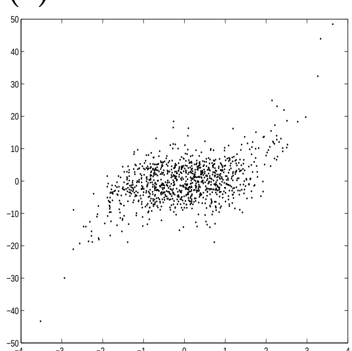

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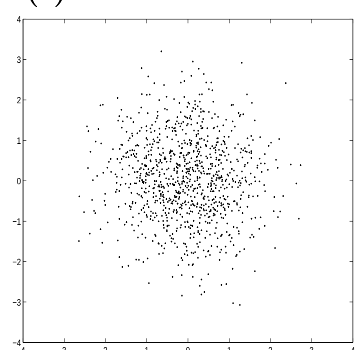

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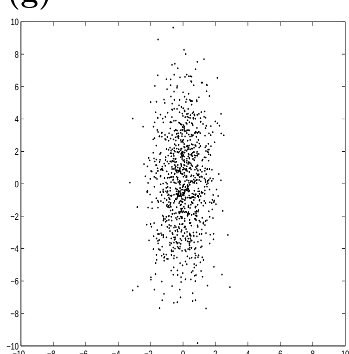

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