

## Assignment 1, CAAM/STAT 581, due September 8

1. Are the following statements true or false? Give proofs/motivations or counterexamples.

a. If  $A$  and  $B$  are countable sets then  $A \cup B$  is countable.

b. If  $A_1, A_2, \dots$  are countable sets then  $\cup_{k=1}^{\infty} A_k$  is countable.

c. The set of irrational numbers is uncountable.

2. Let  $\Omega = (-\infty, \infty)$ . Find  $\cup_{t \in T} A_t$  and  $\cap_{t \in T} A_t$  in the following cases.

a.  $T = \{1, 2, \dots\}$  and  $A_t = (0, 1 + \frac{1}{t})$ .

b.  $T = \{1, 2, \dots\}$  and  $A_t = (0, 1 - \frac{1}{t})$ .

c.  $T = (0, \infty)$  and  $A_t = [e^{-t}, e^t]$ .

3. Let  $\Omega$  be a one-point set,  $\Omega = \{a\}$ , say. Find the power set of  $\Omega$  and the power set of  $2^\Omega$ .

4. Find  $\liminf_n A_n$  and  $\limsup_n A_n$  in the following cases:

a.

$$A_n = \begin{cases} (-1/n, 0) & \text{if } n \text{ is odd} \\ (0, 1/n) & \text{if } n \text{ is even} \end{cases}$$

b.

$$A_n = \begin{cases} (-1 + 1/n, 0] & \text{if } n \text{ is odd} \\ [-1/n, 1 - 1/n) & \text{if } n \text{ is even} \end{cases}$$

c.  $A_n = (-1/n, n)$ .

d.  $A_n$  is the interior (i.e. all points inside the circle but not those on the boundary) of a circle in  $R^2$  with center at  $((-1)^n/n, 0)$  and radius 1.