Assignment 1, CAAM/STAT 581, due September 8

1. Are the following statements true or false? Give proofs/motivations or counterexamples.

a. If A and B are countable sets then $A \cup B$ is countable.

b. If $A_1, A_2, ...$ are countable sets then $\cup_{k=1}^{\infty} A_k$ is countable.

c. The set of irrational numbers is uncountable.

2. Let $\Omega = (-\infty, \infty)$. Find $\bigcup_{t \in T} A_t$ and $\bigcap_{t \in T} A_t$ in the following cases.

a. $T = \{1, 2, ...\}$ and $A_t = (0, 1 + \frac{1}{t})$.

b. $T = \{1, 2, ...\}$ and $A_t = (0, 1 - \frac{1}{t})$.

c. $T = (0, \infty)$ and $A_t = [e^{-t}, e^t]$.

3. Let Ω be a one-point set, $\Omega = \{a\}$, say. Find the power set of Ω and the power set of 2^{Ω}.

4. Find $\liminf_n A_n$ and $\limsup_n A_n$ in the following cases:

a.

$$A_n = \begin{cases} (-1/n, 0) & \text{if } n \text{ is odd} \\ (0, 1/n) & \text{if } n \text{ is even} \end{cases}$$

b.

$$A_n = \begin{cases} (-1 + 1/n, 0] & \text{if } n \text{ is odd} \\ [-1/n, 1 - 1/n) & \text{if } n \text{ is even} \end{cases}$$

c. $A_n = (-1/n, n)$.

d. A_n is the interior (i.e. all points inside the circle but not those on the boundary) of a circle in R^2 with center at $((-1)^n/n, 0)$ and radius 1.