Assignment 4, CAAM/STAT 581, due October 8

1. Let $\Omega = \Omega' = [0, \infty)$ and let $\mathcal{B} = \mathcal{B}' =$ Borel sets.

a. For each of the following functions, find the σ-field it generates.

(i) $h(x) = x$

(ii) $h(x) = x^2$

(iii) $h(x) = 1$ if x is rational, $h(x) = 0$ otherwise

b. Give an example of a function h which is not measurable.

2. Let $\Omega = [0, \infty)$ and let $\mathcal{C}$ be the class of singletons, $\mathcal{C} = \{\{x\}, x \geq 0\}$. Let $\mathcal{A} = \{B \subseteq R : B \text{ is countable or } B^c \text{ is countable}\}$, the so called countable/co-countable σ-field (see book, p.13).

a. Show that $\mathcal{A} = \sigma(\mathcal{C})$.

b. Give an example of a Borel set which is not in $\mathcal{A}$.

c. Let $\Omega' = [0, \infty)$ and let $\mathcal{B}'$ be the Borel σ-field. Which of the following functions are measurable $\mathcal{A}/\mathcal{B}'$?

(i) $h(x) = x$

(iii) $h(x) = x$ if x is rational, $h(x) = 0$ otherwise

(iii) $h(x) = 0$ if x is rational, $h(x) = x$ otherwise

(iv) $h(x) = 1$ if $x \leq 1$, $h(x) = 0$ otherwise