Assignment 5, CAAM/STAT 581, due October 31

1. Let $X_1, X_2, ...$ be i.i.d. random variables having an exponential distribution with mean 1. Show that

a. $P(X_n > \log n \text{ i.o.}) = 1$

b. $P(X_n > n \text{ i.o.}) = 0$.

2. The Hewitt-Savage 0-1 Law does not hold if the X_k are independent but not identically distributed. Give an example of independent $X_1, X_2, ...$ and a symmteric event in $\sigma(X_1, X_2, ...)$ which has probability strictly between 0 and 1. *Hint:* Try to find an event involving the value of the sum $\sum_k X_k$ such that this event is determined by the value of X_1 alone.

3. Let $(\Omega, \mathcal{B}, P) =$ ([0, 1],Borel sets, Lebesgue measure). Define a random variable X on this space such that X has a uniform distribution on $[-1, 1]$, i.e. such that $P(a \le X \le b) = (b - a)/2$ for $-1 \le a \le b \le 1$. Use the definition of expectation to show that $E[X] = 0$.

4. Use Borel-Cantelli to construct examples of sequences of random variables $X_1, X_2, ...$ such that

a. $X_n \to 0$ *a.s.* and $E[X_n] \equiv 1$.

b. $X_n \to 0$ *a.s.*, $E[X_n] < 1$ for all n and $E[X_n] \to 1$.

c. $X_n \to \infty$ *a.s.* and $E[X_n] \to -\infty$.

d. $X_n \not\to 0$ *a.s.* and $E[X_n] \to 0$