Assignment 1, STAT 582, due February 11

1a. Let X be uniform on $[0,1]$, let Y_n be exponential with mean $1/n$ and let $X_n = X + Y_n$. Show that $X_n \to X$ a.s. and in L_1.

b. What is the probability that $X_n \neq X$ infinitely often?

2. Give examples of sequences of random variables $X_1, X_2, ...$ such that

a. $X_n \to 0$ a.s. and in L_1 but not in L_2.

b. $X_n \to 0$ a.s. but X_n does not converge in L_p for any $p > 0$.

3. Let

$$X_n = \begin{cases} n^2 & \text{with probability } 1/2n^2 \\ 0 & \text{with probability } 1 - 1/n^2 \\ -n^2 & \text{with probability } 1/2n^2 \end{cases}$$

Determine if X_n converges almost surely and in L_1.

4. Suppose that $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Use Theorem 6.3.1 (b) (the "subsequence argument") to show that $X_n + Y_n \xrightarrow{P} X + Y$.

5. Let c be a real-valued constant and $X_1, X_2, ...$ a sequence of random variables.

a. Sketch the cdf of c. What does $X_n \xrightarrow{d} c$ mean?

b. Show that if $X_n \xrightarrow{d} c$ then $X_n \xrightarrow{P} c$ (this is a "partial converse" to our general implication chart).