

## Assignment 2, STAT 582, due February 18

**1a.** Let  $X_1, X_2, \dots$  be independent with the same finite mean  $\mu$  and the same variance  $\sigma^2 < \infty$  (but not necessarily i.i.d.). Show that  $S_n/n \xrightarrow{P} \mu$ .

**b.** Now drop the variance assumption: Let  $X_1, X_2, \dots$  be independent with the same finite mean  $\mu$ . Give an example which shows that  $S_n/n \xrightarrow{P} \mu$  may not hold even if  $\text{Var}[X_j] < \infty$  for all  $j$ . *Hint:* Let  $X_j$  be symmetric around 0 in a way such that condition (ii) in Theorem 7.2.1 is violated.

**2.** Let  $X_1, X_2, \dots$  be independent with  $X_j$  having a uniform distribution on  $[0, j]$ . Show that  $S_n/n^2$  converges in probability and determine the limit.

**3.** Let  $X_1, X_2, \dots$  be independent such that  $X_j$  has density  $f_j(x) = c_j/x^3$  for  $x \geq 1/j$ .

**a.** Determine the constants  $c_j$ .

**b.** Let  $S_n = \sum_{j=1}^n X_j$ . Show that  $S_n/\log n$  converges in probability and determine the limit.