

Assignment 2, STAT 582, due February 18

1a. Let X_1, X_2, \dots be independent with the same finite mean μ and the same variance $\sigma^2 < \infty$ (but not necessarily i.i.d.). Show that $S_n/n \xrightarrow{P} \mu$.

b. Now drop the variance assumption: Let X_1, X_2, \dots be independent with the same finite mean μ . Give an example which shows that $S_n/n \xrightarrow{P} \mu$ may not hold even if $\text{Var}[X_j] < \infty$ for all j . *Hint:* Let X_j be symmetric around 0 in a way such that condition (ii) in Theorem 7.2.1 is violated.

2. Let X_1, X_2, \dots be independent with X_j having a uniform distribution on $[0, j]$. Show that S_n/n^2 converges in probability and determine the limit.

3. Let X_1, X_2, \dots be independent such that X_j has density $f_j(x) = c_j/x^3$ for $x \geq 1/j$.

a. Determine the constants c_j .

b. Let $S_n = \sum_{j=1}^n X_j$. Show that $S_n/\log n$ converges in probability and determine the limit.