HW7

15th March 2005

4.3

1. $\lambda = 2/3$ so $\theta = 3/2$ and $\alpha = 10$ Now use the gamma distribution and substitue in for the parameters.

$$f(x) = \frac{1}{\Gamma(10)\frac{3}{2}^{10}} x^9 e^{\frac{-2x}{3}}$$

$M(t) = \frac{1}{(1-\frac{3}{2t})^{10}}$
$E(X) = \alpha\theta = 15$
$Var(X) = \alpha\theta^2 = \frac{45}{2}$

2. Use formula 4.3-1 or you could do integration by parts. Using 4.3-1 with $w = 5$, $\lambda = \frac{1}{4}$ and $\alpha = 2$ you get.

$1 - e^{-5/4} - \frac{5}{4}e^{-5/4} = .35536$

3. We need to find $E(e^{tX})$.

$E(e^{tX}) = \frac{1}{\Gamma(\alpha)\theta^\alpha} \int e^{tx} e^{\frac{-x}{\theta}} x^{\alpha-1} dx$
$= \frac{1}{\Gamma(\alpha)\theta^\alpha} \int e^{-x\frac{(1-\theta t)}{\theta}} x^{\alpha-1} dx$
using hint we let $y = \frac{x(1-\theta t)}{\theta}$ and now $x = \frac{\theta y}{1-\theta t}$ and $dx = \frac{\theta}{1-\theta t}$. We substitute in and get.

$= \frac{1\theta^\alpha}{\Gamma(\alpha)\theta^\alpha(1-\theta t)^\alpha} \int e^{-y} y^{\alpha-1} dy$
but $\Gamma(\alpha) = \int e^{-y} y^{\alpha-1} dy$ since the limits of integration do not change and the above equation becomes

$E(e^{tx}) = \frac{1}{(1-\theta t)^\alpha}$ Note that $1 - \theta t$ must be greater than 0 for the integral to converge.

4. $M(t) = \frac{1}{(1-\theta t)^\alpha}$ $M^{/}(t) = \frac{\alpha\theta}{(1-\theta t)^{\alpha+1}}$ so $M^{/}(0) = E(X) = \alpha\theta$
$M^{//}(t) = \frac{-\alpha\theta(\alpha+1)*(-\theta)}{(1-\theta t)^{\alpha+2}}$, $M^{//}(0) = \alpha^2\theta^2 + \alpha\theta$

$Var(X) = \alpha^2\theta^2 + \alpha\theta^2 - (\alpha\theta)^2 = \alpha\theta^2$

5. From the mgf given
$\theta = 7$ and $\alpha = 20$
$f(x) = \frac{1}{\Gamma(20)7^{20}} x^{19} e^{\frac{-x}{7}}$
$E(X) = \alpha\theta = 140$ *and* $Var(X) = \alpha\theta^2 = 980$

10. Looking at table we find that 5 percent of the $\chi^2(12)$ *distribution is left of 5.226 and 5 percent is right of 21.03.*
$a = 5.226$ *and* $b = 21.03$

14. 5 cars every ten minutes means that $\lambda = .5$ and $\theta = 2$. We are looking for the 8the arrival so $\alpha = 8$ and the pdf is.

$$f(x) = \frac{1}{\Gamma(8)2^8} x^7 e^{\frac{-x}{2}}$$

But we can recognize this as a χ^2 dixtribution with r=16 degrees of freedom. No we look up in the table in the back of the book to find that .

$$P(X > 26.30) = .05$$

15. Since all 15 observations are taken independently from the same distribution. The chance that at most three are greater than 7.779 follows are binomial(n,p) distribution. We just need to find p. p is found by looking in the tables in the back of the book at the $\chi^2(4)$ and is equal to .1 Now we use the binomial distribution($nCx * p^x * (1-p)^{n-x}$) to find the $P(X < 4) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$.
Doing this with p=.1 we get that

$$P(X < 4) = .944$$

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2. a).3078 b) .4959 c).2711 d).1646
e).0526 f).3174 g).0456 h) .0026

4)a)1.28 b) -1.645 c)-1.66 d)-1.82

7) We must make X a standarnd normal random variable first. To do this we let $Z = \frac{x-\mu}{\sigma} = \frac{x-6}{5}$

c)$P(-2 < X < 0) = P(\frac{-8}{5} < Z < \frac{-6}{5}) = .0603$
e)$P(|X-6| < 5) = P(1 < X < 11) = .6826$

10)Using hint we first find $P(Y < y)$
$P(Y < y) = P(aX + b < y) = P(X < \frac{y-b}{a})$
$P(X < \frac{y-b}{a}) = \int \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$ with limits from negative infinity to $\frac{y-b}{a}$. Using hint we will let $w = ax + b$ and $x = \frac{w-b}{a}$ and $dx = \frac{1}{a.}dw$. Substituting in we get

$P(X < \frac{y-b}{a}) = \int \frac{1}{\sqrt{2\pi}\sigma a} e^{-\frac{(\frac{w-b}{a}-\mu)^2}{2\sigma^2}} dw$ the limits of integration are now negative infinity to y. Taking out $\frac{1}{a}$ in the exponent yields.

$P(X < \frac{y-b}{a}) = \int \frac{1}{\sqrt{2\pi}\sigma a} e^{-\frac{(w-(b+a\mu)^2}{2a^2\sigma^2}} dw$. We recognize this as a normal distribution with $\mu = b + a\mu$ and $variance = a^2\sigma^2$or simply put a $N(b + a\mu, a^2\sigma^2)$