HW 9

8th April 2005

5.1-1

a)Sum out all the y's $f(x) = \frac{10+4x}{32} = \frac{5+2x}{16}$
b) Sum up all the x's $f(y) = \frac{3+2y}{32}$
c)P(X>Y)=P(2,1)=$\frac{3}{32}$
d)P(Y=2X)=P(1,2)+P(2,4)=$\frac{3}{32} + \frac{6}{32} = \frac{9}{32}$
e)P(X+Y=3)=P(1,2)+P(2,1)=$\frac{6}{32}$
f)P(X<=3-Y)=P(1,1)+P(1,2)+P(2,1)=$\frac{8}{32}$
g)No P(1,1)=$\frac{2}{32}$ but P(X=1)P(Y=1)=$\frac{14}{32} * \frac{5}{32}$ which are not equivalent and thus X,Y are dependent.

5.1-9

$f(x) = \int f(x,y)dy$ with $0 \leq y < \infty$ which equals $-2e^{-x-y}$ evaluated at 0 and infinity. So $f(x) = -2e^{-x}$

$f(y) = \int f(x,y)dx$ with $0 \leq x \leq y$ This equals $-2e^{-x-y}$ evaulated at 0 and y.
$f(y) = -2e^{-2y} + 2e^{-y} = 2e^{-y}(1 - e^{-y})$

X and Y are not independent. f(x,y) does not equal f(x)f(y).

5.1-10

a)$f(x) = \int (x+y)dy$ where y is in[0,1]
$f(x) = x + \frac{1}{2}$
$f(y)$is computed the same way so $f(y) = y + \frac{1}{2}$
Obviously f(x)f(y) does not equal f(x,y)

b) $\mu_x = \int x(x + \frac{1}{2})dx = \frac{x^3}{3} + \frac{x^2}{4}$ evaluated at 0 and 1 equals $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$
c)$\mu_y = \frac{7}{12}$ by the same reasoning
d)$E(X^2) = \int x^2(x + \frac{1}{2})dx = \frac{x^4}{4} + \frac{x^3}{6} = \frac{5}{12}$
$\sigma_x^2 = \frac{5}{12} - (\frac{7}{12})^2 = \frac{11}{144}$
$\sigma_y^2 = \frac{11}{144}$

5.2-1

$\mu_x = \sum x(\frac{2x+5}{16}) = \frac{7}{16} + \frac{18}{16} = \frac{25}{16}$
$\mu_y = \sum y(\frac{2y+3}{32}) = \frac{5}{32} + \frac{14}{32} + \frac{27}{32} + \frac{44}{32} = \frac{90}{32}$
$E(X^2) = \sum x^2(\frac{2x+5}{16}) = \frac{7}{16} + \frac{36}{16} = \frac{43}{16}$
$\sigma_x^2 = \frac{46}{16} - (\frac{25}{16})^2 = \frac{63}{256}$
$E(Y^2) = \sum y^2(\frac{2y+3}{32}) = \frac{5}{32} + \frac{28}{32} + \frac{81}{32} + \frac{176}{32} = \frac{290}{32}$
$\sigma_y^2 = \frac{290}{32} - (\frac{90}{32})^2 = \frac{295}{256} = 1.15$
$E(XY) = \sum\sum xy(\frac{x+y}{32}) = \frac{2}{32} + \frac{6}{32} + \frac{12}{32} + \frac{20}{32} + \frac{6}{32} + \frac{16}{32} + \frac{30}{32} + \frac{48}{32} = \frac{140}{32} = \frac{35}{8} = 4.375$
$Cov(X,Y) = 4.375 - \frac{25}{16}(\frac{90}{32}) = -.0195$
$\rho = \frac{Cov(x,y)}{\sqrt{\sigma_x^2\sigma_y^2}} = \frac{-.0195}{\sqrt{.2461*1.15}} = -.0367$
This tells us that X and Y are dependent.

6.1-10

a)First compute P(X<85)=$P(Z < \frac{85-75}{10} = 1)$ where Z is a N(0,1) r.v. P(Z<1.5)=.8413. Since all X's are independent and identical.
$P(maxX_i < 85) = P(Z < 1)^4 = .8413^4 = .501$

b) Because of independence this equals $P(60 < X_1 < 80) * P(75 < X_3 < 90) = P(-1.5 < Z < .5) * P(0 < Z < 1.5)$
$= (.6915 - .0668) * (.9332 - .5) = .2706$

c)because of independence and identicall dist $E(X_1^2X_2^2X_3^2) = E(X_1^2)^3 = (100 + 75^2)^3 = 1.876 * 10^{11}$

d)$E(2X_1 + 3X_2 + X_3 + X_4) = E(7X_1) = 7 * 75 = 525$

6.2-5

$E(Y) = -2E(X_1) + E(X_2) = 1$
$Var(Y) = 4Var(X_1) + Var(X_2) = 36 + 25 = 61$

6.2-6

$M_Y(t) = E(e^{tY}) = E(e^{t(X_1+X_2)}) = E(e^{tX_1})E(e^{tX_2})$
Remembering that the moment generatin function for a binomial equals $(1 - p + pe^t)^n$
$M_Y(t) = (1 - p + pe^t)^{(n_1+n_2)}$
This tells us that Y is distributed as a $b(n_1 + n_2, p)$

6.2-7

a)Using logic from problem 6
$M_y(t) = M_{X_1}(t) * M_{X_2}(t) * M_{X_3}(t) = e^{2(e^t-1)}e^{e^t-1}e^{4(e^t-1)}$

$= e^{7(e^t-1)}$

b)Which tells us Y is poisson(7)

c)P(3$\leq Y \leq 9) = .83 - .03 = .8$

6.2-9

$M_y(t) = E(e^{ty}) = E(e^{tX_1})^5$since each X_i has the same value for p.
$M_X(t) = \frac{pe^t}{1-(1-p)e^t}$ since p=1/3 $M_Y(t) = \frac{(pe^t)^5}{(1-(1-p)e^t)^5}$
This is just the MGF of a negative binomial with paramters p= 1/3 and r=5

6.2-10

$M_Y(t) = E(e^{tY}) = E(e^{tX})^h = (\frac{1}{1-\theta t})^h$ This is the MGF of a Gamma(h,θ)
The mean of a gamma is $h\theta$

6.2-11

$M_Y(t) = E(e^{tY}) = E(e^{tX_1})^3 = ((\frac{1}{1-5t})^7)^3 = \frac{1}{(1-5t)^{21}}$
This is distributed as a gamma(21,5)

6.2-14

X is some discrete random distribution that takes on values {0 , 1, 2, ...} and Y is another discrete random distribution that is independent of X and can take on the same values. X is disributed according the function f(x) and Y is distibuted according to g(y).
We now can create a set A={(x,y): x+y=w}

A={(0,w), (1,w-1),...,(w,0)} This set consists of w+1 mutually exclusive events. Now we just need to find the probability of theses w+1 events.

P(x,w-x) is simply f(x)*g(w-x) because of independence. Summing every event in A up will give the h(w) in the text.

$h(w) = \sum f(x)g(w-x)$

6.2-19

X_1 is the random variable where we see how many rolls until we get one new face. Since we have not observed any rolls yet $X_1 = 1$ always. This fits with a geometric(1). X_2 is the r.v where we see how many rolls it will take to get a different face besides the one that we already saw. There is a 3/4 chance that we get a different face than the first roll. We are going to roll until we get that new face. This is equivalent to a geometric(3/4). $X_3$is the r.v where we see how many rolls it takes to see a different face than the 2 other faces we have seen. It is distributed as a geometric(1/2). X_4 is the r.v where we roll till we observe the last face and is distributed as a geometric(1/4). X_2, X_3, X_4 can take on all positive integer values.

b)$E(Y) = E(X_1) + E(X_2) + E(X_3) + E(X_4)$

Since each is is geometric(p) the expected value of a geometric r.v is $\frac{1}{p}$ and its variance is $\frac{1-p}{p^2}$.

$E(Y) = 1 + \frac{4}{3} + 2 + 4 = \frac{25}{3}$

Since the X's are independent, $Var(Y) = Var(X_1) + Var(X_2) + Var(X_3) + Var(X_4)$

$= 0 + \frac{.25}{.75^2} + \frac{.5}{.5^2} + \frac{.75}{.25^2} = \frac{130}{9} = 14.44$

c)To calculate these probabilites we need to look at all the combinations of each X that will add up to the Y. We know that X_1 must equal 1. We then combine this and look at the distribution function for each X, which is $f(x) = (1-p)^{1-x}p$

$P(Y = 4)$ =P(1,1,1,1)=$1 * \frac{3}{4} * \frac{1}{2} * \frac{1}{4} = \frac{3}{32}$

$P(Y = 5) = P(1,1,1,2) + P(1,1,2,1) + P(1,2,1,1) = \frac{144}{1024}$

$P(Y = 6) = P(1,1,1,3)+P(1,1,3,1)+P(1,3,1,1)+P(1,1,2,2)+P(1,2,1,2)+P(1,2,2,1) = \frac{150}{1024}$

$P(Y = 7) = P(1,1,1,4)+P(1,1,4,1)+P(1,4,1,1)+P(1,1,2,3)+P(1,2,1,3)+P(1,2,3,1) + P(1,1,3,2) + P(1,3,2,1) + P(1,3,1,2) + P(2,2,2,2)$

$= \frac{135}{1024}$