HW 11

20th April 2005

7.1

2

This problem is basically done in example 7.1-3 except now we are given μ and just have one parameter to look at. To show it is unbiased we must find $E[\hat{\theta}]$

$E[\hat{\theta}] = E[\frac{1}{n}\sum(X_i - \mu)^2 = \frac{1}{n}\sum E[(X_i - \mu)^2] = \frac{1}{n}\sum Var(X_i) = \frac{1}{n}\sum \sigma^2 = \sigma^2$

9b

$Var(\overline{X}) = Var(\frac{\sum X_i}{n}) = \frac{1}{n^2}Var(\sum X_i) = \frac{1}{n^2}\sum Var(X_i) = \frac{1}{n^2}n\theta^2 = \frac{\theta^2}{n}$

14

We know that $\frac{(n-1)S^2}{\sigma^2}$ is distributed as a chi-squared with n-1 degrees of freedom. Now let $X = \frac{(n-1)S^2}{\sigma^2}$. $X^{\frac{1}{2}}\frac{\sigma}{\sqrt{n-1}} = S$. So, $E[cS] = cE[X^{\frac{1}{2}}\frac{\sigma}{\sqrt{n-1}}] = \frac{c\sigma}{\sqrt{n-1}}E[X^{\frac{1}{2}}]$. Since we know the distribution of X we can do our regular expected value calculation now.

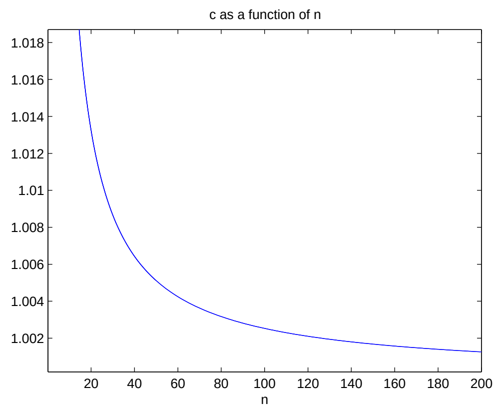
$\frac{c\sigma}{\sqrt{n-1}}E[X^{\frac{1}{2}}] = \int \frac{x^{\frac{1}{2}}x^{\frac{n-1}{2}-1}e^{\frac{-x}{2}}}{\Gamma(\frac{n-1}{2})2^{\frac{n-1}{2}}}dx$ Simplifying this we get $= \frac{c\sigma}{\sqrt{n-1}}\int \frac{x^{\frac{n}{2}-1}e^{\frac{-x}{2}}}{\Gamma(\frac{n-1}{2})2^{\frac{n-1}{2}}}dx$
Now multiply through by the given c.

$= \sigma \int \frac{x^{\frac{n}{2}-1}e^{\frac{-x}{2}}}{\Gamma(\frac{n}{2})2^{\frac{n}{2}}}dx$ But this is just the pdf of a chi-squared(n) and the integral of a pdf is equal to 1. So we are now left with just σ

b) Using matlab

n	
5	1.0638
6	1.0509

c)Plotting the graph we can see that it is converging to 1.



7.2

7

Since sigma is not given we must use s, the sample standard deviation. So our confidence interval would be $\overline{x} \pm \frac{s}{\sqrt{n}} * t_{n-1,.025}$. Here $\overline{x} = 20.9$ $s = 1.8584$ and $t_{8,.025} = 2.306$ and the interval is [19.47, 22.33]

16

Here we are asked to find the expected length of the interval for μ assuming n=5.
a) if σ is known then the interval is simply $E[2 * \frac{\sigma}{\sqrt{n}} z_{.025}]$ Since all of these values are constant this is equal to 1.753σ.

b) if $\sigma$is unknown then the interval is $E[2 * \frac{S}{\sqrt{n}} t_{4,.025}]$ We use the t distribution here because sigma is unknown. We estimate sigma with S.

The expected value is now equal to $\frac{2*t_{4,.025}}{\sqrt{n}} E[S]$ From problem 14 in section 7.1 $E[S] = \frac{\sigma}{c}$ where c is defined in problem 14. In that problem we also calculated c to be 1.0638 when n = 5. Using this we get the interval to equal 2.334σ

8.2

1

We are given that scores come from a normal distribution with $\sigma^2 = 100$. Looking at table 8.2-1, we reject when $\overline{X} \geq \mu_0 + z_\alpha \sigma/\sqrt{n}$ Here $\overline{X} = 113.5$ $\mu_0 = 110$

$\sigma = 10$, and $z_{.05} = 1.645$

a) So the right side of the above equals 114.1 so we fail to reject at the .05 level.

b) At the .1 level the right hand side equals 113.2, so we do reject at this level.

c) The p-value is found by finding the z score which is $\frac{113.5-110}{10/4} = 1.4$ Looking up this number in the tables in the b ack of the book, the p-value is .0808.