Stat 310 Homework 12 Solutions

2nd May 2005

6.4-1

Approximate $P(\frac{1}{2} \leq \overline{X} \leq \frac{2}{3})$ where n=12 and X˜U(0,1)

Using our transformation where $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ ˜ $N(0,1)$

$P(\frac{1}{2} \leq \overline{X} \leq \frac{2}{3})$= $P(\frac{1}{2} - \frac{1}{2} \leq \overline{X} - \frac{1}{2} \leq \frac{2}{3} - \frac{1}{2})$

$= P(\frac{0}{\frac{1}{\sqrt{12}}/\sqrt{12}} \leq \frac{\overline{X}-\frac{1}{2}}{\frac{1}{\sqrt{12}}/\sqrt{12}} \leq \frac{\frac{1}{6}}{\frac{1}{\sqrt{12}}/\sqrt{12}}) = P(0 \leq Z \leq 2)$ where Z is approximately standard normal

$\approx 0.9772 - 0.5 = .4772$

6.4-2

Approximate $P(-0.3 \leq Y \leq 1.5)$ where $Y = X_1 + X_2 + ... + X_{15}$ and all X are iid with $f(x) = (\frac{3}{2})x^2, -1 < x < 1$

Use the fact that $\frac{Y-n\mu}{\sigma * \sqrt{n}}$ is approximately normal.

$\mu = \int_{-1}^{1} \frac{3}{2}x^3 dx = \frac{3}{8}x^4|_{-1}^{1} = 0$ and $\sigma^2 = \int_{-1}^{1} \frac{3}{2}x^4 dx = \frac{3}{10}x^5|_{-1}^{1} = 0.6$

Thus

$P(-0.3 \leq Y \leq 1.5) = P(\frac{-0.3-15*0}{\sqrt{.6}*\sqrt{15}} \leq \frac{Y-15*0}{\sqrt{.6}*\sqrt{15}} \leq \frac{1.5-15*0}{\sqrt{.6}*\sqrt{15}})$

$= P(-0.1 \leq Z \leq 0.5) \approx 0.2313$

6.4-3

Approximate $P(2.5 \leq \overline{X} \leq 4)$when X˜exp(3) and n=36

Using method from 6.4-1

$P(2.5 \leq \overline{X} \leq 4) = P(-1 \leq Z \leq 2) = 0.8185$

6.4-5

$X_1, ..., X_{18}$ are i.i.d. $\chi^2(1)$random variables.

(a) From Example 6.4-5, we know that $\sum_{i=1}^{n} X_i$is a $\chi^2(n)$variable when $X_i$are iid $\chi^2(1)$ variables.

Thus $Y = \sum_{i=1}^{18} X_i$is distributed as a $\chi^2(18)$variable.

(b) We approximate using the same method in 6.4-3

$P(Y \leq 9.390) = P(Z \leq \frac{9.390-18}{\sqrt{2}*\sqrt{18}}) = P(Z \leq -1.435) \approx 0.757$ which is higher than the actual probability of 0.05, and $P(Y \leq 34.80) = P(Z \leq 2.8) \approx 0.9974$ which is only slightly higher than the actual 0.99.

8.1-1

$\alpha = P(H_0 rejected|H_0 true) = P(X = 2or3|p = \frac{1}{3})$

$= P(X = 2|p = \frac{1}{3}) + P(X = 3|p = \frac{1}{3}) = (3c2)(\frac{1}{3})^2(\frac{2}{3}) + (3c3)(\frac{1}{3})^3 = \frac{7}{27}$

$\beta = P(X = 0or1|p = \frac{2}{3}) = (3c0)(\frac{1}{3})^3 + (3c1)(\frac{2}{3})(\frac{1}{3})^2 = \frac{7}{27}.$

8.1-8

$H_0 : p = 0.75$

$H_1 : p < 0.75$

(a) With $\alpha = 0.05$, we will reject when $z = \frac{\hat{p}-9}{\sqrt{\frac{p(1-p)}{n}}} < -1.645$

$z = \frac{0.7-0.75}{\sqrt{\frac{.75*.25}{390}}} = -2.28$

Since $z = -2.28 < -1.645$, we reject H_0

(b) With $\alpha = 0.01$, we will reject when $z < -2.326$

$z = -2.28 > -2.326$, so we fail to reject H_0

(c) The approximate p-value for this test is $P(z < -2.28) = 0.0113$

8.2-3

(a) The test statistic is $z = \frac{\overline{X}-170}{10/\sqrt{25}}$

For $\alpha = 0.05$ the critical region is $\{z : z \geq 1.645\}$

(b) In this case, $\overline{X} = 172.52$ and $z = 1.26$. Since $z < 1.645$, we fail to reject H_0

(c) The p-value is $P(z \geq 1.26) = 0.1038$

8.2-6

(a) $H_0 : \mu = 3.4$

(b)$H_1 : \mu > 3.4$

(c) The test statistic is $t = \frac{\overline{X}-3.4}{s/3}$

(d) For $\alpha = 0.05$, the critical region is $\{t : t \geq 1.86\}$

(e) In this case $t = \frac{3.556-3.4}{0.167/3} = 2.802$

(f)$t = 2.802 > 1.86$, so we reject H_0

(g) The p-value is in between 0.01 and 0.025. If using a computer to find, the p-value is 0.0116.

8.2-11

(a) The test statistic is $\chi^2 = \frac{24*s^2}{(140)^2}$. The critical region is $\chi^2 \leq 36.42$

(b) In this case, $\chi^2 = 29.18$

(c) The confidence interval for μ is approximately
$[0, \overline{X} + \frac{t_{.98}(24)*s}{5}] = [0, 725.432]$

8.4-2

$H_o : p_1 = 0.4, p_2 = 0.2, p_3 = 0.2, p_4 = 0.1, p_5 = 0.1$

We are given $n = 580$, with respective frequencies 224,119,130,48,59. To test, use the following statistic:

$q_{k-1} = \sum \frac{(y_i - n*p_i)^2}{n*p_i}$, which is $\chi^2(4)$

So, $q_4 = \frac{(224-232)^2}{232} + \frac{(119-116)^2}{119} + \frac{(130-116)^2}{116} + \frac{(48-58)^2}{58} + \frac{(59-58)^2}{58} = 3.784$

Since the expected value of q_4 is 4 when H_0 holds, we wil fail to reject H_0 at any reasonable α level

8.4-4

Since there are 3 colors (yellow, white, purple) with respective probabilities $p_1 = \frac{1}{4}, p_2 = \frac{1}{4}, p_3 = \frac{1}{2}$,n=40, and observed numbers are 6, 7, and 27. Our statistic is:

$q_2 = \frac{(6-10)^2}{10} + \frac{(7-10)^2}{10} + \frac{(27-20)^2}{20} = 4.95.$

At an $\alpha = .05$, we would reject is $q_2 > 5.991$. Since $4.95 < 5.991$, we fail to reject the null.

8.4-9

We will use the table from Example 3.5-3 to find our observed frequencies for $A_1, ..., A_{11}$, and then use the Poisson table to find $P(x \in A_i)$ when $\lambda = 5.6$. Then multiply the probabilities by n=100 to find expected frequencies under the null hypothesis that X is a Poisson process. Here are the results to be used:

Frequency	$P(x \in A_i)$	Expected
1	$P(x \leq 1) = 0.024$	$P * 100 = 2.4$
4	$P(x = 2) = .058$	5.8
13	$P(x = 3) = 0.079$	7.9
19	0.151	15.1
17	0.170	17
15	0.158	15.8
9	0.127	12.7
12	0.111	11.1
7	0.055	5.5
2	0.031	3.1
2	$P(x \geq 11) = 0.028$	2.8

The following statistic is used:

$q_{10} = \frac{(1-2.4)^2}{2.4} + ... + \frac{(2-2.8)^2}{2.8} = 6.113$

Since $6.113 < 18.31$, we fail to reject the null hypothesis.

8.5-6

For this problem, we use the following statistic:

$q_{(k-1)*(h-1)} = \sum_{i=1}^{k} \sum_{j=1}^{h} \frac{(Y_{ij} - n(Y_{i.}/n)(Y_{.j}/n))^2}{n(Y_{i.}/n)(Y_{.j}/n)}$

In this case

$q_4 = \frac{(21-100(60/100)(30/100))^2}{100(60/100)(30/100)} + \frac{(5-100(60/100)(8/100))^2}{100(60/100)(8/100)} + ... + \frac{(12-100(40/100)(25/100))^2}{100(40/100)(25/100)}$

$= 8.410$

Since $8.410 < 9.488$,we fail to reject the null hypothesis that sport preference is independent of gender. The p-value is between 0.05 and 0.10. It is approximately 0.78.