STAT 552 Homework 6

Due date: In class on Thursday, Oct 27, 2005

Instructor: Dr. Rudolf Riedi

20. (Extreme values) Let $N(A)$ be a Poisson point process on $\mathbb{R}^+$ with points X_n and with control measure $\mu(A) = \int_A 1/t^2 dt$.

    (a) Let $x > 0$. Show that $\mathbb{E}[N((x, \infty])]$ is finite. Conclude that $P[N((x, \infty]) < \infty] = 1$.

    (b) Let $Y := \sup_n(X_n)$. Argue that the event $\{Y < x\}$ is equal to the event $\{N([x, \infty]) = 0\}$. Using that N is a Poisson process show that $P[Y < x] = \exp(-1/x)$ for $x > 0$. This is a classical extreme value distribution.

    (c) Bonus question (not required, but instructive):
    Show that the transform $T(t) = 1/t$ on the positive real numbers maps the Poisson process N to the *homogenous* Poisson Process $\tilde{N}(\tilde{A}) := N(T^{-1}(\tilde{A}))$. Conclude from the above that the infimum of the points of $\tilde{N}$ has an exponential distribution.
    Notably, one could show that the infimum is actually a minimum; in other words, there is a "first arrival" of $\tilde{N}$, and this first point has an exponential distribution.

21. ($M/G/\infty$ queue)

    Assume that calls are initiated according to a homogeneous Poisson point process with $\{X_n\}_n$ on $(0, \infty)$. Assume that call durations $\{I\}_n$ are i.i.d. with a common distribution G and independent of call initiation times. Let $A(t)$ denote the number of ongoing calls at time t (initiated before t but not terminated at time t). Show that $A(t)$ has a Poisson distribution for every t. Hint: Fix time t and relate $A(t)$ to a marked point process.