STAT 552 Homework 2

Due date: In class on Thursday, September 14, 2006

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5. In a branching process we have

$$P(s) = P_{Z_{j,k}}(s) = as^2 + bs + c \tag{1}$$

with $a > 0$, $b > 0$, $c > 0$ and $P(1) = 1$.

   (a) Compute the extinction probability π.

   (b) Give a condition for sure extinction.

   (c) What is the underlying distribution of offsprings $Z_{j,k}$?

6. Conduct a compound experiment as follows. Suppose, N is a non-negative integer valued random variable with $q_k := P[N = k]$ and $\sum_{k \geq 0} q_k = 1$, i.e., $P[N = \infty] = 0$. Observe N items and mark each of the N items independently of each other and independently of the value of N with a probability p, where $0 < p < 1$. To be precise, set $X_i = 1$ with probability p indicating that the ith item is marked, and $X_i = 0$ otherwise. Let V denote the number of marked items among $X_1, ..., X_N$.

   (a) Relate V to the X_i.

   (b) Give a simple formula for $P_V(s)$.

   (c) What is the probability that all items are marked in terms of p, q_k and $P_N(s) = \sum_{k \geq 0} q_k s^k$?
   Hint: the number of items is random! To avoid problems with "double-randomness" partition the event "all items marked" into sub-events where one the random variables is known. Compare to the derivation of the compound formula in class.

7. Suppose X is a non-negative integer valued random variable with $P[X = \infty] = 0$. Suppose T is a geometric random variable independent of X, i.e., $T \sim g(k; \theta)$ where the parameter θ lies in $(0, 1)$. To be precise, $P[T \geq k] = \theta^k$ for $k \geq 0$.

   (a) Verify that $\sum_k P[T \geq k, X = k] = P_X(\theta)$.

   (b) Compute $P[T \geq X]$.
   Hint: Again, there is "double-randomness". Split the event into sub-events where one the random variables is known.

8. Consider a simple branching process with $P(s) = q + ps$. Let T denote the time of extinction, i.e., $T = \inf\{n \geq 1 : Z_n = 0\}$.

   (a) Express the event $\{T = n\}$ in terms of the events $\{Z_k = 0\}$.

   (b) Express $P[T = n]$ in terms of $P(s)$.

   (c) Derive an explicit formula for $P[T = n]$ in terms of p and q. Hint: recursion.