

# STAT 552 Homework 5

Due date: In class on Thursday, October 19, 2006

Instructor: Dr. Rudolf Riedi

16. Let  $S_n$  denote a simple random walk:  $S_n = X_1 + \dots + X_n$  with  $X_n$  i.i.d. and  $P[X_n = 1] = 1 - P[X_n = -1] = p = 1 - q$ .
- (a) Show that this chain is irreducible, i.e., find for every pair of states  $i, j$  an integer  $n$  such that  $p_{ij}^{(n)} \neq 0$  (the exact value is not needed). Hint: distinguish  $i > j$  and  $i \leq j$ .
  - (b) Based on known results on the return to zero from earlier homework decide whether 0 is recurrent or transient. Hint: your answer will dependent on the parameter  $p$ .
  - (c) Assume  $p > 1/2$ . Use the strong law of large numbers to conclude that  $P[S_n \rightarrow \infty] = 1$ .
  - (d) Assume that 0 is recurrent. Show that  $P[S_n \rightarrow \infty] = 1$  is impossible.
17. Let  $0 \leq s < S$  be two integer parameters. Let  $D_n$  be a sequence of i.i.d. random variables with  $p_k = P[D_n = k] > 0$  for all  $k \geq 0$  and  $P[D_n = \infty] = 0$ . Suppose  $X_0 \leq S$ . Recall that  $(u)_+ = \max(u, 0) = (u + |u|)/2$  and define

$$X_n := \begin{cases} (X_{n-1} - D_n)_+ & \text{if } s < X_{n-1} \leq S, \\ (S - D_n)_+ & \text{if } X_{n-1} \leq s. \end{cases} \quad (1)$$

You may think of  $X_n$  as tracking the stock (number of items) in a store at the end of the  $n$ th day, where  $D_n$  is the demand on the  $n$ -th day. If the stock falls below  $s$  in the evening it is replenished to  $S$  over night.

- (a) Argue in a short sentence that  $X_n$  forms a Markov Chain.
  - (b) Determine its equivalence classes.
  - (c) Compute the long run average stock level  $(X_0 + \dots + X_{N-1})/N$  in terms of the stationary distribution.
  - (d) For the remainder let us consider a specific example. Let  $s = 0$  and  $S = 2$ . Let  $p_0 = 1/2$ ,  $p_1 = 2/5$  and  $p_2 = 1/10$ . Compute the transition matrix  $\mathbf{P}$ .
  - (e) For this example show that the stationary distribution is  $(5/18, 8/18/5/18)$ .
  - (f) For the same example compute the long run fraction of periods of unsatisfied demand, i.e., the long run fraction of days with  $X_n = 0$ .
18. Let  $j$  be an absorbing state of a Markov Chain. Which of the following is true?
- (a) State  $j$  necessarily transient,
  - (b) state  $j$  necessarily recurrent,
  - (c) state  $j$  could be either.

If your answer is one of the first two options, provide an argument; if your answer is the third option, provide two Chains, one with an absorbing transient state and one with an absorbing recurrent state.