

ELEC 535 Homework 4

Due date: In class on Friday, February 21, 2003

Instructor: Rudolf Riedi Rice University, Spring 2003

Problem 4.1 (Optimal Codes)

- (a) Find some binary ($D = 2$) prefix free code for the source with probabilities $(1/3, 1/5, 1/5, 2/15, 2/15)$.
- (b) Find the optimal prefix free code, i.e., the one with minimal expected length. Hint: optimize the expected length under the boundary condition of the Kraft (in)equality for this source and construct a code with the resulting optimal code lengths.
- (c) Is this optimal code also optimal for the source with probabilities $(1/5, 1/5, 1/5, 1/5, 1/5)$? Show your reasoning. Hint: any prefix free code has to satisfy the Kraft inequality.

Problem 4.2 (The game of Hi-Lo)

- (a) A computer generates a number X according to a known (in particular also known to the player) probability mass function $p(x)$, $x \in \{1, 2, \dots, 100\}$. The player asks a question, "Is $X = i$?" and is told "Yes", "You're too high", or "You're too low." He continues for a total of six questions. If he is right (i.e. he receives the answer "Yes") during his sequence, he receives a prize of value $v(X)$ (which is independent of how long it takes to find the answer). How should the player proceed to maximize his expected winnings?
- (b) The above doesn't have much to do with information theory. Consider the following variation: $X \sim p(x)$, prize = $v(x)$, $p(x)$ known, as before. But *arbitrary* Yes-No questions are asked sequentially (not only "higher-or-lower-than" questions) until X is determined. Questions cost now one unit each. Show that the expected return is $\sum_x p(x)(v(x) - l(x))$, where $l(x)$ is the number of questions required to determine the object x and depends on the strategy of the player.
- (c) Maximize the above expected return to find the optimal strategy $l(x)$.