ELEC 697: Homework Oct. 1, 1998

Consider the experiment of continued coin tossing. Let X_n be the outcome of the n-th toss, i.e. X_n =0 for heads, $X_n = 1$ for tails, with equal probability.

Let F_n be the smallest sigma algebra which makes $X_1, X_2, ...,$ and X_n measurable, i.e.

the sets $\{X_k < a\}$ are in F_n for all $k = 1, .., n$ and all real a. (*)

1) Give the four sets (events) which constitute F_1.

2) How many sets (events) are in F_2?

Let $S_n = X_1 + ... + X_n$

3) Show by using the definition (*) that S_2 is F_2 measurable, but not F_1 measurable.

4) Give 2^n 'simple events' (events which are determined uniquely by the first n tosses) which form a 'basis' for F_n. How many sets (events) are in F_n?

Let G_n be the smallest sigma algebra which makes S_n measurable.

5) Give the $n+1$ events which form a 'basis' for G_n. (HINT: S_n can take $n+1$ different values). How many elements (ω-s) do these 'basic events' contain (not all contain the same number). Show that G_n consists of exactly 2^{n+1} events.

6) Conclude from the analysis done in 5) that G_n is smaller than F_n, but that it contains events outside of F_{n-1}. This proves that S_n is F_n measurable, but not F_{n-1} measurable, generalizing 3).

7) Compute (by conjecturing a formula and verifying it using the definition of conditional expectation)

$$E[S_5|F_2]$$
$$E[S_5|F_3]$$
$$E[S_5|F_4]$$
$$E[S_5|F_5]$$
$$E[S_5|F_6]$$

8) Compute

$$E[X_1 + X_3|F_1]$$
$$E[X_1 + X_3|F_2]$$
$$E[X_1 + X_3|F_3]$$
$$E[X_1 + X_3|F_4]$$

9) Compute

$$E[X_5|F_2]$$
$$E[X_5|F_9]$$

For hints email riedi@rice.edu .