ELEC 697: Homework Sept. 22, 1998

1) Proof the following:

If for some $a > 1$ the limit

$$\lim_{n\to\infty} \frac{N_{a^{-n-1}}(E)}{N_{a^{-n}}(E)} = b$$

exists, then $d_{\text{box}}(E) = \log b / \log a$.

To do so, proceed in two steps:

a) If for some $a > 1$ the limit $\lim_{n\to\infty} N_{a^{-n+1}}/N_{a^{-n}} = b$ exists, then

$$\lim_{n\to\infty} \frac{\log N_{a^{-n}}(E)}{n \log a} = \log b / \log a.$$

b) If $\log N_{a^{-n}}(E)/\log a^n \to \log b / \log a$, then

$$\lim_{\delta\to 0} \frac{\log N_{\delta}(E)}{-\log \delta} = \log b / \log a.$$

As an application have the Cantor set in mind ($a = 3$, $b = 2$).

2) Let p be any positive number. The set $E = \{n^{-p} : n = 1, 2, ...\}$ is countable. Its Hausdorff dimension is, therefore, 0. Compute its box dimension.

Hint: No hidden tricks required. Use straightforward counting.

For hints email riedi@rice.edu .